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THE DEVELOPMENT OF STUDENTS' THINKING USING COLORED CARDS TOWARDS INTERSECTION AND UNION CONCEPT

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Abstract. *Intersection and union concepts are crucial part of operation in sets, especially in solving word problems and creating the venn diagram. In fact, not a few students do fallacies in representing sets word problems into a sets operation, so they do mistakes in solving the problems. This study aims to investigate students' thinking in learning intersection and union concept, specifically by using colored cards. The learning is designed under Realistic Mathematics Education (RME) approach and the principle used is emergent modeling. The method used in this study is design research. The subjects of this study is 25 students of the 7th grade junior high school, in which five of them were the focused group. The main findings in this study are description of students' thinking while learning union and intersection using colored cards in the class, and any kinds of students' 'diagram venn'models.*

Keywords: *design research, intersection, realistic mathematics education, union, venn diagram*

Sets is a branch of mathematics that lives in our daily life. It actually has been existed since the mathematics itself had being developed. But, a German mathematician, Georg Cantor came and started to study deeper about its theories (Ferreiros, 2007). Nowadays, set theories have become crucial part of mathematics, and its fundamental starts to be learned since junior high school. Education in Indonesia gives sets as a material in mathematics lesson that learned when a student starts study in junior high school.

Based on several mathematics education researches in Indonesia, known that sets is a mathematics material in junior high school that quite hard for most of the students. Sabil's study (2013) in one of schools in Jambi found that students didn't pass standard score criteria given, 60. Moreover, Asnidar (2014) showed in her study in a junior high school in Palu that plenty of students do wrongs in solving sets question specially word problem. From 30 students, about 57,78 % didn't do well. Armana et al., (2011) based on their study in Surakarta stated that percentage of students whom able to classify objects by sets characteristic is 17,9%, percentage of students that can distinguish which group is sets and which group is not sets is around20,5%, percentage of students that able to restate sets concept is 12,5%. Those numbers are small considering sets and its concept is closely applicable in our dailylife.

Beside that, Darma et al. (2013) in their study stated a fact that in Indonesia the steps of learning mathematics in school that commonly teachers do are: 1) giving theory/ definition/ formulas; 2) giving examples of question; 3) giving questions to practice. It makes us wonder how actually the process of students' thinking in learning sets this way, considering mathematics is abstract.

To prevent mathematics' abstract, Gravemeijer (2004) stated that a rather common strategy in mathematics education is to try to concretize abstract, formal, mathematical concepts in concrete manipulative or visual schemas. The idea then is that these abstract concepts are made easier to grasp by making them

concrete. Freudenthal (in Van-Den Heuvel Panhuizen, 2003) stated that there is no mathematics without mathematizing.

This study uses Realistic Mathematics Education (RME) as the approach of learning sets. Freudenthal (2002) clarified that RME fosters the attitude of experiencing mathematics as a human activity. It means students are active to do activities in order to find the concepts by themselves. Since that, RME provides and shows mathematizing process of students' learning. Beside using context as one of tenets of RME, Gravemeijer (2004) said the idea is that the students not only mathematize contextual problems —to make them accessible for a mathematical approach— but also mathematize their own mathematical activity, which brings their mathematical activity at a higher level (Gravemeijer, 2004). Hence, using RME as the approach of a learning is suitable in order to see the development of students' thinking.

Based on the problems explained above, we need to know that how do exactly students' thinking when they are learning sets in the class. To get specific answer, we ought to limit and to look centered at two main operations of sets, intersection andunion. One possible method to merge students' way of thinking in learning union and intersection is through changing the method of learning sets in class. An approach which pushes students to creatively think and to emerge their thinking by mathematizing is Realistic MathematicsEducation (RME).

The objective of this study is to investigate how is students' way of thinking in learning union and intersection usingRME.

METHOD

This study uses design research as a method of the research and descriptive qualitative as the approach. As described in chapter 1, this study aims to investigate how students' way of thinking in learning union and intersection using RME. For this purpose, design research was chosen as an appropriate means to answer the research question and get the research goal. Van Erdee (2013) stated that design research in mathematics education aims at understanding the process of learning and teaching mathematics. Gravemeijer and Cobb (Van Erdee, 2013) clarified that the main aim of design research is to develop instruction theories about the learning of students and to develop educational materials that are designed to support the learning. In consonance with Gravemeijer and Cobb, Van Erdee (2013) argued that design research offers teacher a unique opportunity on learning to understand students' thinking and learning. Hence, design research is the right research method to answer the question of this study.

Van Erdee (2013) stated 3 phases in design research:

(1) Phase 1: Preparation anddesign
In this phase, researcher started by collecting more knowledge (K) for the research: doing a literature review, the formulation of research aim and the general research question. After that, researchers designed (D) Hypothetical Learning Trajectory, student worksheet, and teacher guide.

(2) Phase 2: TeachingExperiment
Then, researchers did teaching experiment (E) in the class room using HLT, student worksheet, and teacher guide made before. The teaching experiment was done with 25 students of the 7^{th} grade junior high school, and five students of them were the focused group. This experiment used 3 camera, in which 2 static cameras (one camera specifically for recording focused group activities, the other one to record the whole class activities), 1 dynamic camera handled by a partner to take video or picture of important parts when the teaching experiment being done, and one recorder to record student discussion in focused group precisely.

(3) Phase 3: RetrospectiveAnalysis
Once the data are collected they have

to be prepared for the retrospective analysis, for example transcribing video and audio registrations (Van Erdee, 2013). In this phase, researchers used the HLT as a guide of the analysis to reflect (R) the teaching experiment. During the analysis the hypothesized learning, the assumptions about students' learning are compared with the actual learning as observed during the lessons.

Design research has a cyclic character. Van Erdee (2013) said that the cyclic character can also relate to micro cycles. Such a cycle only refers to a set of problems and activities during one lesson. This following figure shows some micro cycles.

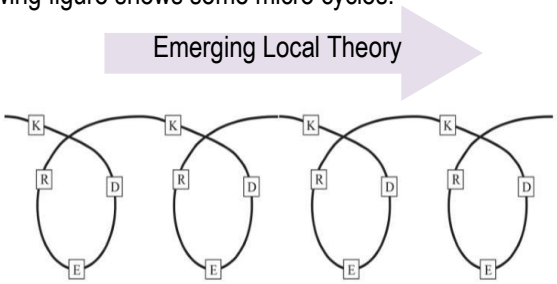


Figure 1. Cyclic Process of Knowledge, Design, Experiment, Reflection And (New) Knowledge

In this study, we just used one cycle, like explained before. As explained in introduction, the learning was designed with Realistic Mathematics Education (RME) as the approach. This study used colored cards as the context, which used to be the situation in grouping things.

On the other hand, Gravemeijer (2004) characterized Realistic Mathematics Education (RME) by three design heuristics: guided reinvention, didactical phenomenology, and emergent modeling. This study utilized emergent modeling as the principle in designing the sequence of activitieslearning.

In emergent modeling, Gravemeijer (1994) stated that instead of trying to concretize abstract mathematical knowledge, the objective of emergent modeling is to try to help students model their own informal mathematical activity. As for the process of emerging the model, Gravemeijer (1994) provided four levels of emergent modeling, those are:

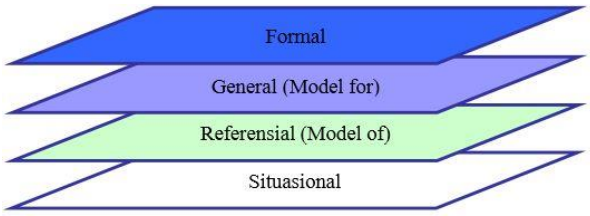


Figure 2. Four Levels In Emergent Modeling

Furthermore, Gravemeijer (1994) explained that figure as follows:

(1) Situational level
Activity in the task setting, in which interpretations and solutions depend on understanding of how to act in the setting (often in out-of-school settings). In this study, situational level shown when students gathered with their friend whom had the same color of the cards they got. So, they were grouping themselves according to the color of the cards theyhad.

(2) Referential level
Referential activity, in which models-of refer to activity in the setting described in instructional activities

(mostly posed in school).
After they grouped themselves, they drew the situation of the position on a worksheet. It means they were making a model of situation.

(3) General level
General activity, in which models-for enable a focus on interpretations and solutions independently of situation-specific imagery;
In this level, students transferred the model of situation they'd been made to solve word problems of sets. The model they made used for another context.

(4) Formallevel
Reasoning with conventional symbolizations, which is no longer dependent on the support of models-for mathematicalactivity. In this level, the students' model of situation was changed to be more formal.

RESULTS AND DISCUSSION

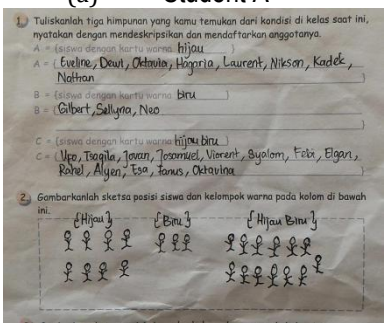
As explained in Chapter 1 and 2, researchers used colored cards as the context in the learning. Before the learning started, the students had learned about sets and ways to state a set. First of all, students sat in their each group and researcher gave each studentworksheet.

Researcher shuffled the cards, which contained of 30 green colored cards and 30 blue colored cards. Then, the cards were shared to the students, each student got two cards. There was 3 eventualities of the cards they got: 1) they got two green cards, 2) they got two blue cards, and 3) they got one green and one blue card. After that, they who got the same combination of cards' color, gathered while brought their worksheet and stationary. Beside that, researcher gave a piece of rope for each group to restrict their group withothers.

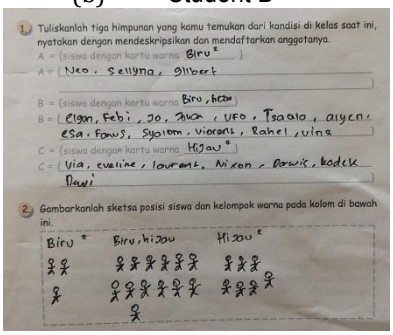
Figure 3. Students Are Gathering with Whom Got the Same Colored Cards (Situtional Level)

After they had been in the group, they answered the questions number 1 and 2 in worksheet. It aimed to take note of the students' position. Here are some answers.

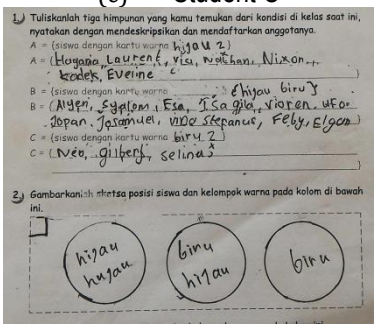
(a) Student A



(b) Student B



(c) Student C



(d) Student D

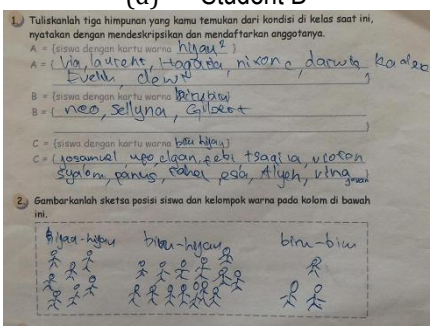


Figure 4. Students' Model of Situation

Based on the answers above, we know that students could make a model from the situation. They changed the situation of student grouped and stated it in set, using a letter for naming it, using bracket in order to distinguish sets and not sets, and stating sets by description and also by listing themembers.

Furthermore, they made a model of situation like a diagram Venn, eventhough they'd not learned it before. Student A and student D looked it by position of groups. They didn't notice about how many members are there in each groups, they focused on the position of the groups and their restrictions. Onething interesting that makes their answers different is the shape of the restrictions. Student A used rectangles, while student D usedcircles.

The other three students gave more concern in the number of member in each set. They didn't make any restrictions to limit one to each other. They had differences in numbering the member of it, student B wrote there were 8 people in group students whom got two cards were green, but the other two wrote it as 7. It's because an error happened while counting and wrote in

number one. Nevertheless, onething that makes their answer different is naming the group. Student B and C named the sets student whom got two cards are green or two cards were blue, as green or blue. It means they looked into the color it self the students had. Beside that, student E named the sets of students whom got two cards were green or two cards are blue, as green-green or blue-blue. It means this student concerned at the combinations of color that students got, not thecolor.

Then, from the three ropes students had, researcher take one rope, and instructed students, "I take one rope you have, so all of you only have two ropes. Try to restrict this three group by using those two rope. You can't cut this rope". Students started to think any way to to do so. They discuss and practice any ways. Some way the did are:

(1) They took the longer rope and crossed it so it could restrict 2 groups.

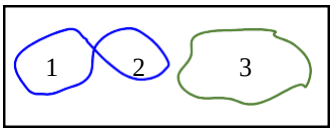


Figure 5. Students' Model Of Situation Of Intersection 1

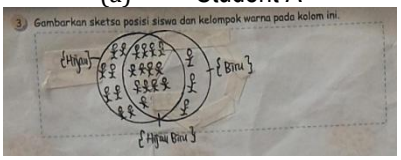
Researcher told them that the way was less precise, "Not to make one rope can restrict 2 groups, but you have to used those two ropes to restrict these threegroups".

(2) The students tied up each rope, it can be shaped like closed curve. Then, they crossed that two tied ropes. The illustration in the class in in this figure.

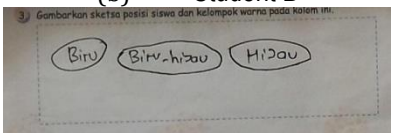
Figure 6. Students' Model Of Situation Of Intersection 2

Researcher appreciated the students' thinking, and let them draw their finding in worksheet. These are some question they had:

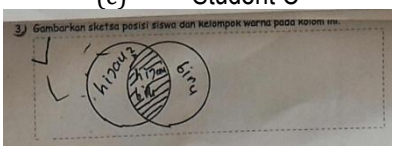
(a) Student A



(b) Student B



(c) Student C



(d) Student D

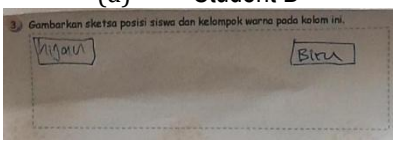


Figure 7. Students' Model Of Intersection

Conforming to the answers of student above, two of five students (B and D) answered correctly and as expected. But, student B's answer is quite closer to real diagram venn of intersection because by noticing it we can determine the number of members in each sets.

After that, in the worksheet, every students wrote the name of the students whom got the blue card and whom got the green card separately. Next, they circled the names that written twice. The names written twice was called the intersection of the sets. Then, students by themselves wrote their understanding about intersection and draw and shade the model of the intersection follows the last context.

Table 1. Students' Understanding About Intersection

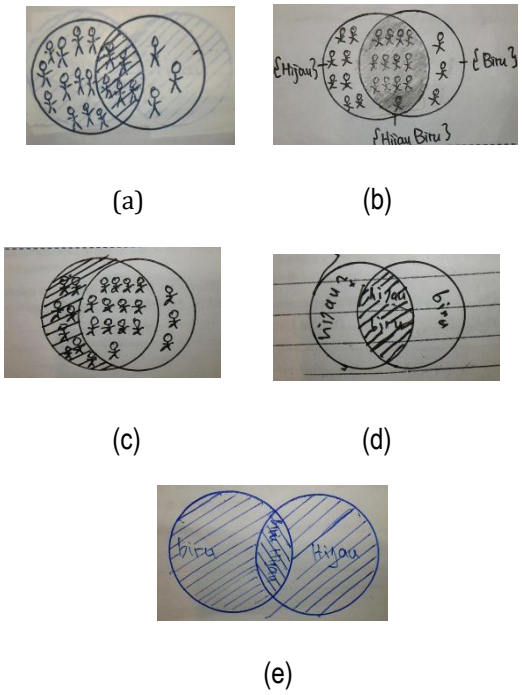
Student	**Answers**
A	Intersection of two sets is a set lays between those two sets
B	Intersection of two sets is a set which the members are mutual members of two sets
C	Intersection of two sets is a set that has two parts
D	Intersection of two sets is a set that exist in those group.

Source: Private Document

Based on students' answers, two of them, A and E had the same answer, they wrote that intersection of two sets is a set that exists in the group. In line with further interview with them, the sentence means if there are two sets, a set that canbe made from that sets is the intersection. Student B said that intersection of two sets is a set lays between those two sets. It means, the position of the set is exactly between those two sets if we look at the model drawn. Student C said that intersection of two sets is a set that has two parts. This student meant intersection has a part in first set and a part in second set. That's why she wrote two parts. The last, is the most closest to the real meaning of intersection of two sets. Students C answered that Intersection of

two sets is a set that the members are mutual members of two sets.

After giving a definition for intersection of two sets, at last they drew a model of intersection using the recently condition if colored cards sharing.



(a) (b)

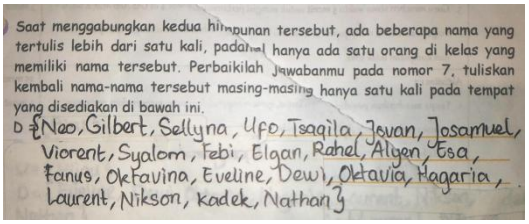
(c) (d)

(e)

Figure 8. Students' Last Model Of Intersection

After that, they walked to the activity for union concept. First, researcher asked them to rewrite all of the members of set students with blue card and all of the members of set students with green card become one set. Then, they were asked, "is there any name that written twice?". Most of them answered, "yes".

Next, they started to think why do the name written twice while the person whom has the name was only one. Most of them, rewrite the combined sets with only write once if there is same part (figure 8).


Saat menggabungkan kedua himpunan tersebut, ada beberapa nama yang tertulis lebih dari satu kali, padahal hanya ada satu orang di kelas yang memiliki nama tersebut. Perbaikilah jawabanmu pada nomor 7, tuliskan kembali nama-nama tersebut masing-masing hanya satu kali pada tempat yang disediakan di bawah ini.

D {Neo, Gilbert, Sellyna, Ufo, Tsaqila, Jovan, Josamuel, Viorent, Syalom, Febi, Elgan, Rahel, Alyen, Esa, Fanus, Oktavina, Eveline, Dewi, Oktavia, Hagaria, Laurent, Nikson, kadek, Nathan}

Figure 9. Combining Two Groups with No Repeated Members

After that, students answered what union of two sets is. Student B answered that union of two sets is a set which the members contain of members of blue and members of green. Student C answered union of two sets is a combining of two sets and the members contains of each set. Student D said that union of two sets is a set that has two same part. By notice the answers, student B didn't understand the question and still focused to the recently situation, student C and D understood the question but couldn't able to deliver theanswer. Students A and E didn't answer the question. Those mean, to sum up the definition of union of two sets is still quite confusing, so the researcher had to give more confirmation at the end of learning.

CONCLUSIONS

Based on the results and discussion described in chapter 3, the development of students' thinking using colored cards towards union and intersection marginally written as:

(1) Students were in situational level, using colored cards shared, they used their prior knowlegde to group defined things.

(2) Students thought about how to connect two groups which has mutual members.

(3) Students were in referential level, using two ropes, making model of situation, and drew it on a paper.

(4) Students defined intersection of two sets bythemselves.

(5) Students combined two groups which have mutual friends and eliminated the members that writtentwice.

(6) Students rewrite the sets of combined two groups which have mutual friends with no members writtentwice.

(7) Students defined union of two sets by themselves.

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