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Numerical Study of Fusion Cross Sections for $^{12}C + {}^{12}C$ and $^{16}O + {}^{16}O$ Reactions by Using Wong Formula

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ABSTRACT−The height of the barrier between the interacting nuclei is one of the interesting topics in nuclear reaction, especially in charged-particle nuclear reactions. Wong formula is one method that can be used to perform such study, especially about fusion cross section. Therefore, a study about fusion cross sections of some light nuclei with Wong formula becomes very interesting to be performed. In this study, the fusion cross sections of $^{12}C + {}^{12}C$ and $^{16}O + {}^{16}O$ reactions have been calculated by using Wong formula at $12 \leq E \leq 32$ MeV of energies. The potential of the interacting nuclei was approached by using Woods-Saxon potential. The calculations performed numerically by using both finite difference and Nelder-Mead methods. The obtained results of this study have achieved a good agreement with the experimental results and the calculation results of the other researchers. Those results were indicated that Wong formula has good capability in explaining the experimental results concerning fusion cross section of light nuclei.

KEYWORDS : *Wong formula, fusion reaction, cross section, light nuclei*

I. INTRODUCTION

The height of the barrier between the interacting nuclei is one of the interesting topics that should be concerned in order to study about nuclear reaction, especially in charged-particle nuclear reactions. By a measurement, it can provide information on the fusion process. It is an important intermediate step to produce super-heavy nuclei by heavy-ion reactions. Wong formula, which was first proposed by Wong (Wong 1973), is one method that can be used to perform such study, especially to explain the experimental results of fusion cross section. Therefore, a study about fusion cross sections of some light nuclei with Wong formula becomes very interesting to be performed.

In this study, it was calculated the fusion cross section of $^{12}C + {}^{12}C$ and $^{16}O + {}^{16}O$ reactions at $12 \leq E \leq 32$ MeV of energies by using Wong formula. The energies were selected based on the experimental data. The potential of the interacting nuclei has been approached by using Woods-Saxon potential. The obtained results have been also compared with the experimental results obtained by Kolata *et al.*(Kolata*et al.* 1980), Fernandez *et al.*(Fernandez*et al.* 1978), and Sperr *et al.*(Sperr*et al.* 1976). It is also compared with calculation results of Sperr *et al.*(Sperr*et al.* 1976) and Kondo*et al.*(Kondo*et al.* 1989), who used the optical model, and with calculation results of Simenel *et al.*(Simenel*et al.* 2013), who worked with the density constrained time-dependent Hartree-Fock (DC-TDHF) method.

In this study, it is also investigated the capability of Wong formula in explaining the experimental results. It is very important information as a fundamental reference for further study about fusion cross section using heavy and super-heavy nuclei. It is also very useful to provide fusion cross section data for unavailable experiment data or the difficulty in cross section measurement by experiments.

In using Wong formula, it should be considered first that the interacting nuclei are spherical and without dynamical distortion. Next, the various barriers are approximated for different partial waves by inverted harmonic-oscillator potentials of height E_l and frequency ω_l. For an energy E, the probability $P(l, E)$ for the absorption of the l-th partial wave is then given by the Hill-Wheeler formula (Wong 1973)

$$P_{E,l} = \left\{1 + \exp\left(\frac{2\pi(V_B - E)}{\hbar\omega_l}\right)\right\}^{-1} \quad (1)$$

where $\hbar$ is the reduced Planck constant and V_B represents the height of barrier potential. The total potential can be calculated by using

$$V_{tot}(r) = V_c(r) + V_N(r) \quad (2)$$

where

$$V_c(r) = \frac{Z_p Z_T e^2}{r} \quad (3)$$

and

$$V_N(r) = \frac{-V_0}{\left[1 + \exp\left(\frac{r - R_0}{a}\right)\right]} \quad (4)$$

where Z_p and Z_T represent the atomic number of the projectile and the target respectively,e is the electron charge,r is the distance between the interacting nuclei,V_0 represents the potential depth, R_0 represents the radius, and a represents the surface diffuseness parameter. The V_0 and R_0 can be estimated by using the following approximations (Akyuz 1979; Gao *et al.* 2014)

$$V_0 = 16\pi\gamma\bar{R}a \quad (5)$$

$$R_0 = r_0\left(A_p^{1/3} + A_T^{1/3}\right) \quad (6)$$

$$R_{T(p)} = 1.23A_{T(p)}^{1/3} - 0.98A_{T(p)}^{-1/3} \quad (7)$$

$$\bar{R} = \frac{R_T R_p}{R_T + R_p} \quad (8)$$

$$\gamma = \gamma_0\left[1 - k\left(\frac{(N_T - Z_T)(N_p - Z_p)}{A_T A_p}\right)\right] \quad (9)$$

where r_0 represents the distance parameter and the subscript T and p represent the target and the projectile respectively. The constants of k and γ_0 were set respectively at 1.8 and 0.95 (in $\mathrm{MeV \cdot fm^{-2}}$) (Akyuz 1979). The parameter of $\hbar\omega_l$ (where $\hbar\omega_l \simeq \hbar\omega_B$), represents the characterizing the behavior of the fusion cross section at very low energy near and below the Coulomb barrier, which can be approached by

$$\hbar\omega_B = \hbar\left[\frac{1}{\mu}\frac{d^2V(r)}{dr^2}\bigg|_{R_B}\right]^{1/2} \quad (10)$$

The reaction cross section can be calculated by using the following formula (Wong 1973; Hagino and Takigawa 2012)

$$\sigma_F(E) = \frac{\hbar\omega_B R_B^2}{2E}\ln\left\{1 + \exp\left(\frac{2\pi(E - V_B)}{\hbar\omega_B}\right)\right\} \quad (11)$$

where R_B is the barrier distance. The parameter V_B is obtained as the peak of total potential. All quantities used in this study were set in atomic unit.

II. METHOD

Calculation process in this study was performed numerically. The numerical differential as shown in Eq.((**10**) was calculated by using finite difference method, which can be formulated as (Chapra 2012)

$$\frac{d^2y}{dx^2} = \frac{y(x + \Delta x) - 2y + y(x - \Delta x)}{(\Delta x)^2} \quad (12)$$

where Δx represents the step of the axis. The parameters a and r_0,shown in Eq. ((**4**) and Eq.((**6**), were optimized by using Nelder-Mead method. This method is very useful to optimize computational problems by using the numerical method or to solve the analytic problems with an unknown gradient (Mathews and Fink 1999). The capability of this method has been proven successfully in calculating the function with two variation parameters (Mathews and Fink 1999; Yulianto and Su'ud 2016).

The step of this method is not so complicated. For example, consider $f(x, y)$ as function. The first step, it is taken three trial coordinates, i.e.(x_1, y_1), (x_2, y_2), and (x_3, y_3). After calculated, those coordinates are noted as follows: $B = (x, y)$ as the best vertex (vertex with minimum value), $G = (x, y)$ as a good vertex, and $W = (x, y)$ as the worst vertex. Next step, the function is optimized by following the algorithm explained in Table 1. Many examples in optimizing the functions can be found in the following reference (Mathewsand Fink 1999).

Table 1 The algorithm of Nelder-Mead method (Mathewsand Fink 1999)

Case (i)	
Determine vertex as B, G, and W	
Compute	
$M = (B+G)/2$, $R = 2M - W$, and $E = 2R - M$	
If $f(R) < f(G)$, then	
Perform case (ii) → either reflect or extend	
Else	
Perform case (iii) → either contract or shrink	
Case (ii)	Case (iii)
Begin	Begin
If $f(B) < f(R)$ then	If $f(B) < f(W)$ then
Replace W with R	Replace W with R
Else	Compute $C = (W+M)/2$
Compute E and $f(E)$	or $C = (R+M)/2$ and $f(C)$
If $f(E) < f(B)$ then	If $f(C) < f(W)$ then
Replace W with E	Replace W with C
Else	Else
Replace W with R	Compute S and $f(S)$
End if	Replace W with S
	Replace G with M
End if	End if
End Case (ii)	End Case (iii)

To ensure the precision of calculation, chi-square distribution can be utilized as long as the experimental data can be extracted. The chi-square can be determined by using (Koonin 1990; Al-Ghamdi and Ibraheem, 2016)

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\left(\sigma_i^{\text{theory}} - \sigma_i^{\text{exp}}\right)}{\Delta\sigma_i^{\text{exp}}}\right]^2 \qquad (13)$$

where $\Delta\sigma_i^{\text{exp}}$ represents the uncertainty in measurements and N is the number of data. The minimum value of χ^2 measures the quality of the fit. The smaller the value ofχ^2, the higher the quality of the fit (Koonin 1990).

III. Results and Discussion

In this study, the parameters of a and r_0 were first optimized by using the Nelder-Mead method. In this process, the parameters were optimized in order to get minimum chi-square of fusion cross section for each reaction. To guarantee the saturation of calculation,the chi-square tolerance was set at 10^{-20}.

The optimized parameters and the calculation results of some parameters for each reaction are shown inTable 2. From those results, it can be indicated that the barrier potential and barrier distance increase with the increase of nucleon number of the interacting nuclei.Special for$^{16}O + ^{16}O$ reaction, it is needed further investigation to reduce the value ofχ^2.

Table 2 Calculation results of $^{12}C + ^{12}C$ and $^{16}O + ^{16}O$ reactions.

Parameters	Reactions	
	$^{12}C + ^{12}C$	$^{16}O + ^{16}O$
a (fm)	0.61984	0.62282
r_0 (fm)	1.01031	1.01437
V_0 (MeV)	35.3400	40.3064
R_0 (fm)	4.6261	5.1121
V_B (MeV)	6.6621	11.4610
R_B (fm)	7.1	7.3
$\hbar\omega_B$ (MeV)	2.9897	3.44086
χ^2	0.2897	2.3387

Next step, it is calculated the fusion cross sections for $^{12}C + ^{12}C$ and $^{16}O + ^{16}O$ reactions by using Wong formula at $12 \le E \le 32$ MeV of energies.The potential models of each reaction have been displayed in Figure 1 and Figure 2.

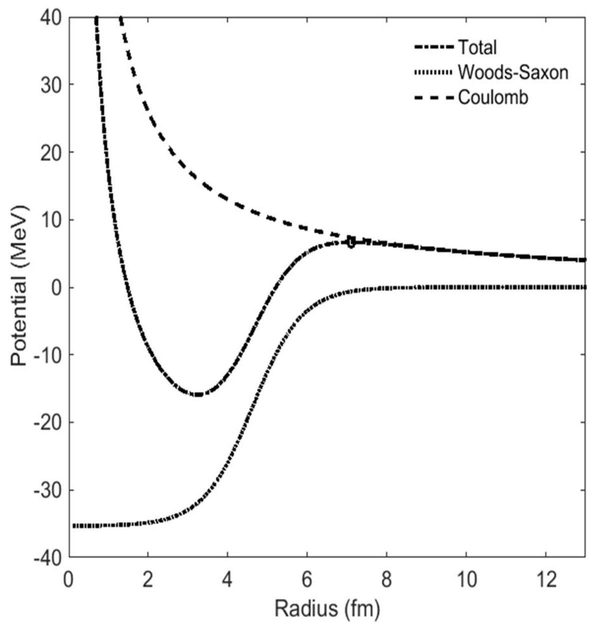

Figure 1 The potential model for $^{12}C + ^{12}C$ reaction

From those figures, it can be seen that each reaction has a similar pattern of potential. At distance above 6 fm, Woods-Saxon potential goes in approaching zero and so does the Coulomb potential smoothly.

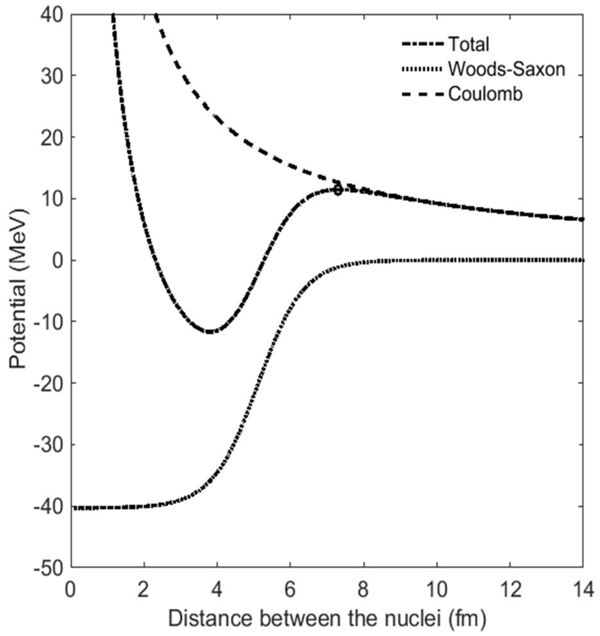

Figure 2 The potential model for $^{16}O + ^{16}O$ reaction

Finally, for the cross section of each reaction observed in this study, it can be seen inFigure 3 and Figure 4.The obtained results of this study have been also compared to the experimental results obtained by previous researchers.

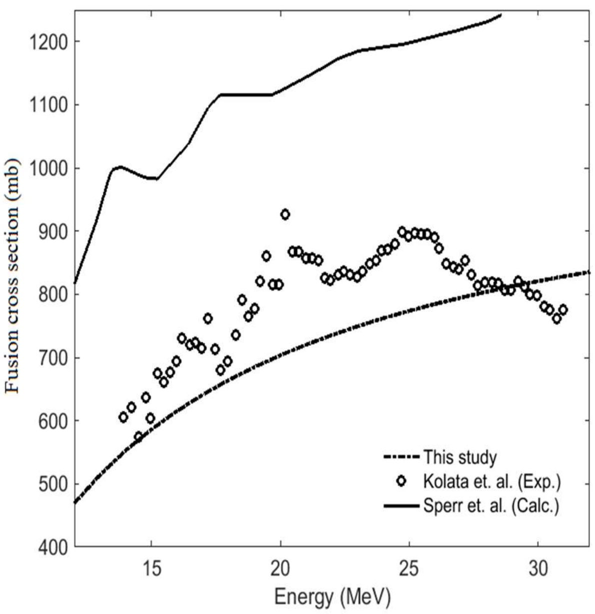

Figure 3 The fusion cross section for $^{12}C + ^{12}C$ reaction, obtained by Kolata *et al.*(Kolata*et al.* 1980), Sperr *et al.*(Sperr *et al.*1976), and this study

InFigure 3, it is displayed the results of this study for $^{12}C + ^{12}C$ reaction compared with the experimental results obtained by Kolata *et al.* and calculation results obtained by Sperr *et al.*It can be seen that the calculation results of this study are in excellent agreement with the experimental results obtained by Kolata *et al.*On the other hand, further investigations should be performed, especially for 20-25 MeV and above 30 MeV, because there are big gaps between this study results and the experimental results at those energies.

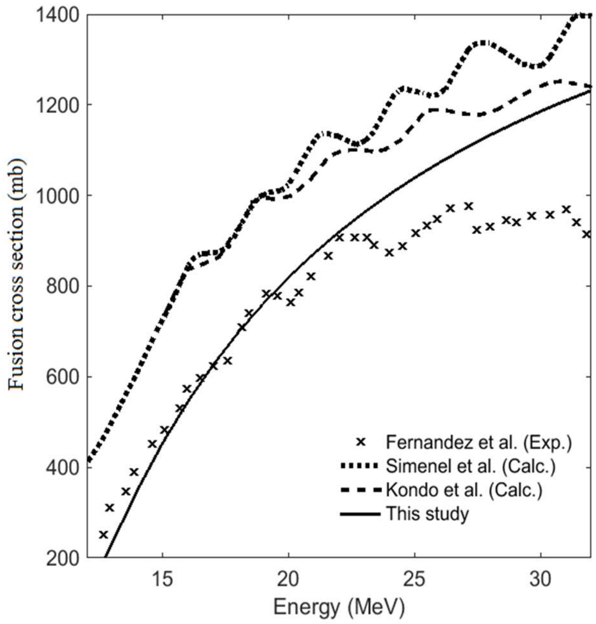

Figure 4 The fusion cross section for $^{16}O + ^{16}O$ reaction, obtained by Fernandez *et al.*(Fernandez*et al.* 1978), Simenel *et al.*(Simenel*et al.* 2013), Kondo*et al.*(Kondo *et al.* 1989), and this study

Figure 4shows the calculation results of this study for $^{16}O + ^{16}O$ reaction compared with the results of other researchers. Good agreement between this study results and the experimental results is achieved for this reaction. The patterns of the lines among the obtained results are similar to each other.In this reaction, the discrepancies between this study results and the experimental results are also found at above 22 MeV of energy. It needs further investigation also to explain those discrepancies.

Referring those results above, it can be indicated that Wong formula used in this study has good capability to explain the experimental results of fusion reaction. It can be a useful tool in explaining the experimental results or in predicting fusion cross section of light nuclei in order to provide the nuclear data.

IV. SUMMARY

A study about fusion cross sections of some light nuclei has been performed successfully. The fusion cross sections of $^{12}C + {}^{12}C$and $^{16}O + {}^{16}O$ reactions have been calculated by using Wong formula at $12 \leq E \leq 32$ MeV of energies. The potential of interacting nuclei has been approached by using Woods-Saxon potential. The parameters of reaction have been optimized by using Nelder-Mead method.

The obtained results of this study have achieved a good agreement with the experimental results and the calculation results of the other researchers. Those results were indicated that Wong formula has good capability in explaining the experimental results concerning fusion cross section of light nuclei.

V. ACKNOWLEDGMENT

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