The Basic Concepts of ModellingRailway Track Systems usingConventional and Finite Element Methods

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Abstract - *The classical concepts of railway track analysis, such Beam on Elastic Foundation (BOEF), Winkler's theory or Zimmermann method are categorized as one-dimensional analysis of a railway structure and are simplification of a beam laid on a continuous support (soil's subgrade or foundation). These methods are still very useful for analyzing a simple design and analysis of railway track systems. Unfortunately, for doing a complex analysis of a railway track, these methods have lack of capabilities, since they only take into account one-dimensional system and neglect the actual discrete support provided by crossed sleeper, ballast, sub ballast mat and subgrade.Nowadays, the use of computer software for doing Finite Element Method (FEM) or Finite Element Analysis (FEA) of a structure is very common for engineers. FEA consists of a huge amount of complex calculations; therefore, a manual calculation by hand is almost impossible to be done. Hence, the use of computer software will be very useful in this manner. The applications of FEM using software also widen in the field of railway infrastructure design and analysis. There are many advantages of using FEM method using computer. However, related to its complexities, one should understands the concepts and "knows-how" to solve the problems, to idealize the structure into FEM model in computer, and to choose the suitable elements and its behaviours, and also the correct method.Thispaper is presented to discuss the basic theories behind the conventional and advanced ways of modelling of railway track system, to show the basic concepts of modelling railway track systems using FEM, to present two- and three-dimensional FEM models of railway superstructures which are built using software ANSYS, and to demonstrate the way of doing the verification of the results using Zimmermann method.*

Keywords: BOEF, Finite Element Method (FEM), Railway Track, ANSYS

INTRODUCTION

Nowadays, the utilization of computer software for modelling or designing a construction is very common for engineers. Computer offers a wide range of capabilities for doing complex tasks in the field of structure analysis, for instance for doing simulation and modelling, investigating the behaviours of a structure and analyzing different scenarios of design.

Finite Element Method (FEM) or Finite Element Analysis (FEA) is a method of material and structure analysis, which at the beginning is developed and commonly used to investigate physical and mechanical behaviours of materials. Due to the fact that this method consists of a huge amount of complex calculations, a manual calculation by hand is almost impossible to be done. This method is then developed into the use of computer software to do the calculations of FEM.

The implementations also widen into the field of railway infrastructure design. The classical concepts of railway track analysis, such Beam on Elastic Foundation (BOEF), Winkler's theory or Zimmermann method are categorized as one-dimensional analysis of a railway structure and are simplification of a beam laid on a continuous support (soil subgrade or foundation). Unfortunately, in a complex simulation, a railway track system cannot be

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simply idealized as one-dimensional system. Researchers also need to investigate the behaviours of changing the material properties of railway components, such as elasticity of rail pad. Furthermore, they sometimes also need to investigate the resulted stresses, deflections and forces in different locations of railway track' components. In this situation, the conventional methods of calculation are not sufficient enough to have the capabilities for doing these tasks. Fortunately, FEM analysis by using computer software is a powerful tool to deal with these problems.

This research is conducted to discuss the basic theories behindthe traditional and advanced ways of modelling of railway track system, to show the basic concepts of modelling railway track systems using FEM, to present two- and three-dimensional FEM models of railway superstructures which are built using software ANSYS, and to demonstrate the way of doing the verification of the results using Zimmermann method.

LITERATURE REVIEW

Beam on Elastic Foundation (BOEF)

As quoted by Cai and Raymond (1994) from Kenney (1954), Fryba (1972), Kerr (1972), Patil (1988) and Duffy (1990), conventional studies of rail track dynamics were a simplification of the interconnected track/beam system as merely Bernoulli-Euler type beam (rail) on an elastic (Winkler type) foundation, or BOEF. In the railway application, in the concept of Winkler support model, the elements of conventional track are basically modelled as two parallel continuous beams (the rails), which are constrained at regular intervals (space) of sleepers. Then these sleepers are assumed have no deformation because they are supported from below and from the sides by ballast bed. Meanwhile, the ballast bed also cannot be deformed. Winkler's hypothesis was that at each point of support the compressive stress is proportional to the local compression (as described by Esveld, 2001). This can be illustrated Figure 1 below:

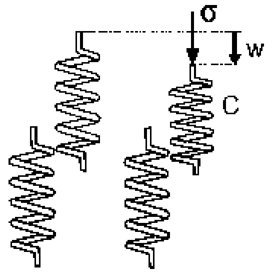

Figure 1. Winkler support model.[1]

The relation can be formulated as:

$\sigma = C.w$

where:

σ = local compressive stress on the support [N/m²]

C= foundation modulus [N/m^3]

w= local subsidence of the support [m]

Sadeghi and Barati (2010) stated that some real conditions of railway tracks are neglected in this approach, such as actual discrete support provided by cross sleepers, interaction between support materials (i.e. ballast, sub-ballast, and subgrade materials), different track supporting layers are not clearly distinguished (in Winkler's method, track support is considered as a one-layer component) and Winkler's model assumed that supporting sleepers fastened tightly to the rail would resist against rail bending through their rotational stiffness. However, best credit is given to Winkler's approach, which was the pioneer of the concept influence line of deflection on the rail on elastic foundation.

In the 1880s, Zimmermann developed a method to determine the forces and deflections which occur in a single supported track loaded by trains in his book "Die Berechnung des Eisenbahnoberbaues" (Steidl, 2007 and Kurrer, 2008). This method was based on Winkler's theory of elasticity and strength. In this theory the rail is considered as a long beam continuously supported on an elastic system. The basic

[1]Source: Esveld, C. (2001), "Modern Railway Track". Second Edition", MRT Production, http://www.esveld.com/MRT_Selection.pdf, last accessed: 26.02.2011

idea in Zimmermann method is to transform the single supported beam by transferring the bearing areas into a continuously supported beam. This method then improved by Eisenmann (Steidl, 2007). This method is illustrated in this figure:

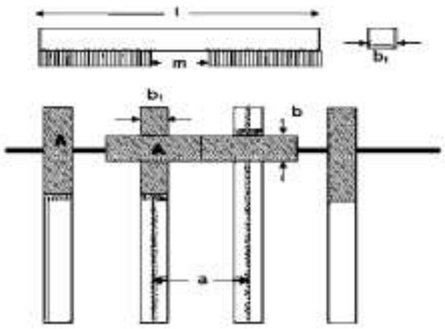

Figure 2.The concept of Zimmermann's theory.[2]

where:

l = length of sleeper

m = length of area without support

b_1 = width of sleeper

The supported areas F = (l - m) * b_1/2 are transformed by connecting the support areas of adjacent sleepers to come with a theoretical continuously supported rail. The length of transformed area is the sleeper space a. Thus, the width of transformed area is b = F/a.

In the Zimmermann method, the single value of C (N/mm^3) or modulus sub-grade reaction or ballast module is used. Meanwhile, in the reality, in ballasted track systems, the components of rail-pad, ballast, sub-ballast mat and sub soil have different C values. Hence, the material properties of those components should be combined into single C_{tot} value by using this correlation:

$$\frac{1}{C_{tot}} = \frac{1}{C_{rail-pad}} + \frac{1}{C_{ballast}} + \frac{1}{C_{sub-ballast-mat}} + \frac{1}{C_{sub-soil}}$$

If the property of material is presented by k value (spring coefficient), then into Zimmermann, k can be converted to C by using this correlation:

$$C = \frac{k}{a.b}$$

The characteristic length of this longitudinal structure is determined by this equation:

$$L = \sqrt[4]{\frac{4.E.I}{b.C}}$$

Zimmermann method enables to calculate deflections and bending moments in several locations by using an influence factor of deflection (η) and influence factor of bending moment (μ):

$$\eta = \frac{\sin\xi + \cos\xi}{e^{\xi}}, \quad \text{and} \quad \mu = \frac{-\sin\xi + \cos\xi}{e^{\xi}},$$

where $\xi = \frac{x}{L}$, and x is the distance between point of interest and the location of the load, while L is the characteristic length.

Therefore, the deflection line and moment diagram can be defined by:

$y = \frac{Q}{2.b.C.L}.\eta$, and $M = \frac{Q.L}{4}\mu$, where Q is the static load applied on the top of the rail.

Finally, the bending stress in the middle of the rail is:

$\sigma = \frac{M}{Wx}$, where Wx the section modulus (static moment) of the rail is.

The rail deflection y activates the contact pressure between rail and sleeper. This contact pressure gives a rail seat load:

$$S = b.a.C.y$$

Discrete Rail Support

Regarding some actual factors which are not considered in Winkler's model, some researchers developed a further modeling approaches which are taken into account the condition of discrete support of rail.

The concept of discrete rail support is illustrated in the Figure 3 below:

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[2]Steidl, Michael (2007), Standards and Test of Fastening Systems.Conference and Proceeding 2007. Arema.org. http://www.arema.org/eseries/scriptcontent/custom/e_arema/library/2007_Conference_Proceedings/Standards_and_Tests-Fastening_Systems_2007.pdf, last accessed: 26.02.2011

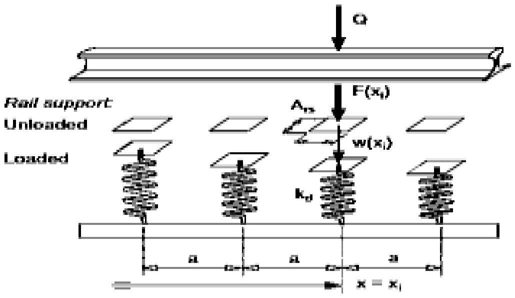

Figure 3.Discrete elastic support model.[3]

As explained by Esveld (2001), the formulation of discrete rail support can be described as follow:

1. According to Winkler, between the vertical force $F(x_i)$ on a support number at $x = x_i$ with effective rail support area A_{rs} and the deflection $w(x_i)$, the following relation exists:

   $$F(x_i) = CA_{rs}w(x_i) = k_d w(x_i)$$

2. Hence the spring constant of the support is: $k_d = CA_{rs}$
3. Determining the spring constant in a railway track with a homogeneous support is relatively simple using the equilibrium condition:

   $$k_d = \frac{\Sigma F}{\Sigma w} = \frac{Q}{\Sigma w}$$

In a further detail of modeling rail on a discrete support, according to Cai and Raymond (1994), the idealized rail track/beam system can be modelled as it is illustrated in Figure 4.a and b. They explained that the vertical dynamic track model considers a conventional ballasted sleeper track, where either Bernoulli-Euler or the Timoshenko beam theory might be applied in both the rail and the sleeper. Through the coupling spring/damper elements representing the resilience and damping of the rail pads and rail-fastening mechanism, it is assumed that the rail is periodically coupled at discrete points (sleeper space) to the cross track sleeper beam. What is more, an axial force in the rail beam is considered to simulate thermal forces. To take into account concrete sleeper beams that have deeper shoulder sections, the sleeper beam can be non-uniform as well, as shown in Figure 4.b. Meanwhile, the elasticity and damping effect of the track foundation (ballast and subgrade) are represented by the distributed spring/damper constants beneath each sleeper. They also considered about possibility of uneven ballast/subgrade compaction efforts across the track which can be included in this model by defining the distributed stiffness/damping coefficient beneath the center portion of the sleeper beam to be different from (always lower than) that beneath the two end segments of the sleeper.

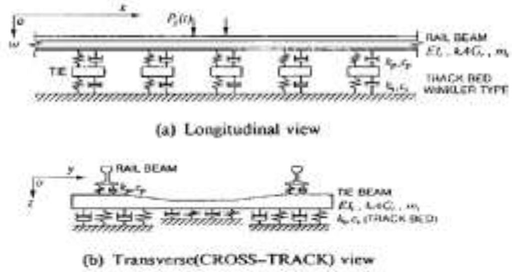

Figure 4. The idealized rail track/beam system.[4]

Finite Element Method

As summarized from Madenci and Guven (2006); Suvo and Khemani (2010); Liu and Quek (2003); and Moaveni (1999) about Finite Element Method (FEM) or Finite Element Analysis (FEA): Nowadays, it becomes a powerful computational method to approximate solutions of a variety of "real-world" practical engineering problems, which have complex domains subjected to general boundary conditions. The basis of FEA relies on the division of the problem domain into a finite number of subdomains (elements). Then, known physical laws are applied to each element, which usually has a very simple geometry. As the result, FEA reduces the problem complexity by solving matrix equations (also so called interpolation functions) of

[3]Source: Esveld, C. (2001), "Modern Railway Track". Second Edition", MRT Production, http://www.esveld.com/MRT_Selection.pdf, last accessed: 26.02.2011

[4]Source: Cai, Z and G.P. Raymond (1994), "Modelling the Dynamic Response of Railway Track to Wheel/Rail Impact Loading", http://civil.queensu.ca/people/faculty/raymond/Notes/845RailCourseNotes/mTrackD1.pdf, last accessed: 26.02.2011

each element by iteration at specific points, referred to as nodes. With respect to the further development and wider area of application of this method, in a complex and detail analysis, the amount of the equations to be solved is usually so large, so that obtaining solution without using computer is practically almost impossible. Therefore, the need of using FEM software packages is necessary.

One popular and wide-used FEM software package is software ANSYS. As stated in the presentation from ANSYS, Inc. (2008), the accurate results of FEM modeling can be achieved by taking carefully some aspects, namely:

1. Understanding the physics of the problem,
2. Understanding the behavior of the elements,
3. Choosing the correct element, the number of elements, and their distribution,
4. Critically evaluating the results and making modification in the conceptual model to improve their accuracy,
5. Understanding the effects of the simplifications and assumptions used.

A common problem which often occurs is the difficulties to achieve convergence simulation. This might be caused by two main factors (Wang, 2004):

1. FE model is not idealized correctly in a physical sense,
2. FE model is not presented correctly in a numerical sense (bad conditioned FE model).

METHODOLOGY

FEM Model 2D in ANSYS

In the two-dimensional basic FEM model in ANSYS, the components of track infrastructures are modelled as a rail beam supported by springs. The illustration of beam supported by springs as a mass-spring system can be seen in this following figure:

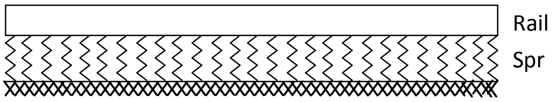

Figure 5. Idealization of rail track using mass-spring systems

The ANSYS's elements which are used to build this two-dimensional basic model are: BEAM3 and COMBIN14.

FEM Model 3D in ANSYS

ANSYS's elements SOLID65, SOLID185 are used to model the components of railway infrastructure in three dimensions. Steel rail, sleepers, ballast are idealized using SOLID65. Meanwhile, rail pads are modelled using SOLID185, which has hyperelastic capability. The interfaces between these elements are connected by using contact elements.

This FEM modelis used for basic rail design, which can be categorized as macro model, because the point of interest of research is laid on the behaviors of the whole system (especially the stresses and deflections on the rail).Thus the multi-layer system of ballast which is modelled using element SOLID65 is reliable enough to handle three-dimensional behaviors of railway track's components. The connection between two surfaces of those components is provided by contact element pairs, namely contact elements CONTA175 with target element TARGE170 in ANSYS.

The aim of modeling all components by using solid element is to have the same degrees of freedom (DOF). To achieve a convergence simulation, it is recommended to use element with the same DOF, especially if contact element is also used. Although in ANSYS it is possible to couple elements with different DOF (e.g. rail is idealized using 6-DOF-element BEAM188) by using a contact element, since the analysis will be a static analysis and the results of simulation will be focused on the stresses and deflections, therefore, the three DOFs offered by solid element is considered already sufficient. Moreover this will speed up the running time of simulation and reduce the risk of divergence of analysis and ill-conditioning matrices.

RESULT

Verification of 2D FEM Model using Zimmermann Method

In the 2D model, a spring constant value of k = 5000 N/mm is simply taken as an input of COMBIN14. The k value represents the combination of each spring constant of rail-pad, ballast, sub-ballast mat and sub soil. This value is only a simple example to do verification of the basic model and is not based on actual data or empirical data. At one of the end of the spring, which is not connected to the rail, a set of boundary conditions is defined. These boundary conditions are that there are no translations and rotations in all directions. The result of deflection line diagram of ANSYS can be seen in this following figure:

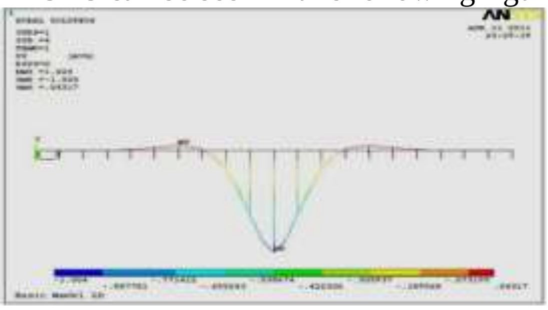

Figure 6. Graphical result ANSYS: contour of deflection line of basic model 2D.

For the given value of spring constant k = 50000 N/mm in the FEM model, by using example values of sleeper space a = 600 mm (ballasted track system) and length of sleeper l = 2600 mm, width of sleeper b_1 = 260 mm and length of unsupported area is assumed to be m = 500 mm, then the value of C = 0.183 N/mm^3 is used in the Zimmermann calculation. This following figure shows the comparison of deflection results between the basic 2D FEM model and manual calculation using Zimmermann method:

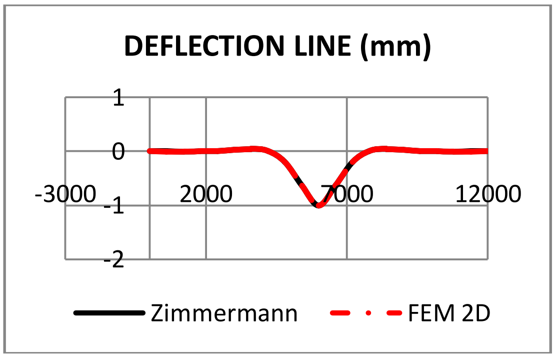

Figure 7.Deflection results comparison between basic model FEM 2D and Zimmermann method.

From the chart above, it can be visualized that the result of deflection line of FEM ANSYS is similar with that of Zimmermann method. It is also proved by using t-student test to check the similarity of both results, which is shown by t-student test value of 99.73%.

Verification of 3D FEM Model using Zimmermann Method

All solid materials are assumed isotropic, which it means that they have the same properties in all directions. Rail-pads, sleepers and ballast are modelled using solid element, which has input of material properties of Young's modulus (E) and Poisson's ratio. Meanwhile, in the Zimmermann method, single value of ballast module or sub-grade reaction (C) is used. Thus, the inputs of E in each FEM element should be converted into C in Zimmermann method. The conversion can be defined by using the relation between Young's modulus of materials and Hooke's Theory of mass-spring systems:

Young's Modulus:

$$E = \frac{Stress}{Strain} = \frac{F/A}{\Delta L/L}, therefore \Rightarrow F = \frac{E.A.\Delta L}{L} = \left(\frac{E.A}{L}\right).\Delta L$$

Hooke's Theory:

$$F = k.x$$

Combining those theories, where x = ΔL:

$$k = \frac{E.A}{L}$$

, where L here is the height/thickness of track's components, hence

$$k = \frac{E.A}{h}$$

Where k in complete track' systems, where among components are in series each other:

$$\frac{1}{k} = \frac{1}{k_{rail-pad}} + \frac{1}{k_{ballast}} + \frac{1}{k_{sub-ballast-mat}} + \frac{1}{k_{sub-soil}}$$

In the 3D FEM models, it is assumed that the sleepers provide full support, which means that all contact areas between sleepers and ballast are fully "in-contact". This contact area is equal with the bottom surface area of the sleeper. Thus in the Zimmermann method, the length of area without support (m) is also assumed equal with zero (full support). After several trials, it is found that changing the value of m does not change significantly the result of deflection in Zimmermann method, hence the assumption either of using m = 0 here or m = 500 as it is in verification 2D does not matter.

The half of 3D basic model (single rail) can be modelled in cross-sectional direction as a spring systems as show in this figure:

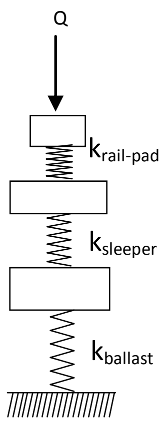

Figure 8. Idealization of basic model 3D into mass-spring system

The total spring constant system (k_{sys}) is a combination of a spring rail-pad which is in series with a spring sleeper and spring ballast. Thus k_{sys} can be defined by this formula:

$$\frac{1}{k_{sys}} = \frac{1}{k_{rail-pad}} + \frac{1}{k_{sleeper}} + \frac{1}{k_{ballast}}$$

The track's length in the model 3D is 12260 mm and sleeper space is 600 mm, thus, there will be 21 sets of rail-pads and sleepers' spring systems, then k_{sys} is:

$$\frac{1}{k_{sys}} = \frac{21}{k_{rail-pad}} + \frac{21}{k_{sleeper}} + \frac{1}{k_{ballast}}$$ and

$$C = \frac{k_{sys}}{a.b}$$

By using example values of sleeper space a = 600 mm (ballasted track system) and length of sleeper l = 2600 mm, width of sleeper b_1 = 260 mm and length of unsupported area m = 0 mm, and the material properties of: rail pad ZW687a (k=500 kN/mm), sleeper B70 (Concrete C30/40 MPa) and ballast bed of crushed stones (120 N/mm^2), thus k_{sys} = 23.67 kN/mm is obtained and C = 0.07 N/mm^3 in Zimmermann method is used.

The resulting deflections of basic FEM model 3D can be seen in the following figure:

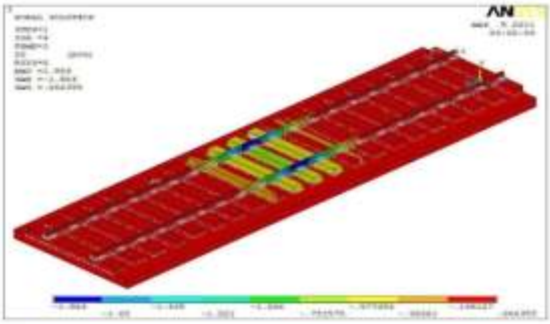

Figure 9. Result of deflections of 3D Basic Model

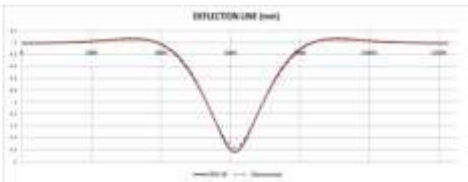

Figure 10.Deflection results comparison between basic model FEM 3D and Zimmermann method.

To compare the results between 3D FEM model and Zimmermann, the statistical t-student test is used to determine whether two results are likely similar. For the comparison between 3D FEM basic model and Zimmermann, the t-student test value is 99.39%. From the Figure 10, it can be seen that in 3D FEM Model, there is a slightly different result of maximum deflection. This may be caused by the geometry of element and the boundary systems of the model which is in three dimensions, while in

Zimmermann the system is considered in one dimension. Furthermore, the concept of Zimmermann's method is a rail on a continuous support, meanwhile in FEM 3D rail lies on discrete support. In addition, in FEM 3D, the Poisson's ratio of material is also considered. Furthermore, the set small stiffness on contact elements of FEM 3D might also have slightly influence to the result of deflections.

CONCLUSION

This study is addressed to give a brief introduction of the concepts of classical and FEA modelling of railway track systems. The two- and three dimensional FEM models using software ANSYS have been presented in this paper. The comparison and verification of the results using manual calculation of Zimmermann method has been also carried out.

The verification shows that in the two-dimensional FEM model, the result of deflection line of FEM ANSYS is very similar with that of Zimmermann method. Meanwhile, in the three-dimensional FEM model, there is slightly different result of deflections between FEM and the manual calculation using Zimmermann method, especially the maximum deflection. Some factors caused this are: (1) the different of geometry of element and the boundary systems between one- (Zimmermann) and three- dimensional (FEM) systems; (2) the different concepts between a rail on a continuous support (Zimmermann) and 3D rail lies on discrete support (FEM); (3) the Poisson's ratio of material is taken into account in FEM; and (4) the behaviours of contact elements used in 3D FEM model.

The utilization of software computer for doing FEM simulations of railway track systems is very useful. ANSYS is a very powerful tool to do this and offers robust library for doing simulation. In the one hand, this is a challenge and a possibility of using strong tools for doing simulation closer to the reality, but on the other hand, sometimes it is confusing for beginners. The common mistakes are that the lack of one's understands especially of the behaviours, properties of materials and element models, and idealizations used in the model leads to a fatal mistake of the results. Therefore a well prepared planning, adequate background knowledge of material properties and behaviours and how to model it, the procedure of analysis, and verification and validation should be done step by step and systematically to obtain an optimal results.

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