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# SOFT GROUPOID AND ITS PROPERTIES

**Saman Abdurrahman**

Department of Mathematics, Lambung Mangkurat University, Indonesia
Faculty of Mathematics and Natural Sciences, Banjarbaru, South Kalimantan, Indonesia
email: saman@ulm.ac.id

## ABSTRACT

A groupoid is a generalized form of the concept of a group, achieved by omitting the properties of associativity, identity, and inverses. In this paper, we introduce the concept of a soft groupoid, which serves as a generalization of the soft group. We define and explore the properties of intersection, AND, and union on soft groupoids and soft subgroupoids. Furthermore, we explore the properties of these operations when applied to collections of soft subgroupoids derived from a given soft subgroupoid.

**Keywords**: Groupoid, soft groupoid, soft subgroupoid

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## INTRODUCTION

Soft groupoid is a generalization of the soft group concept, which has been introduced by several authors, including (Abdurrahman *et al*., 2024; Aktaş & Çağman, 2007; Alajlan & Alghamdi, 2023; Barzegar *et al*., 2023; Çağman *et al*., 2012; Kaygisiz, 2012; Yin & Liao, 2013), similar to how groupoids generalize groups in classical theory. A soft set over a groupoid $S$ is represented as $\sigma_S \overset{\text{def}}{=} \{(a, \sigma(a)) \mid a \in A, \sigma(a) \subseteq S\}$, where $\sigma$ is a function from the parameter set $A$ to the power set $P(S)$. In more detail, a soft set $\sigma_S$ defined over a group $S$ is referred to as a soft group over S if $\sigma(a)$ is a subgroup of $S$ for every $a \in A$. The concept of soft group was introduced by (Ghosh *et al*., 2016; Oguz, G., Icen, I. ve Gürsoy, 2020; Oguz, 2023; Voigt, 2022). This concept has wide applications in rough set theory, fuzzy sets, and other fields involving uncertainty or data complexity.

As an extension of the group, soft groupoid introduces a framework for working with elements in a soft set, where these elements may have uncertain relationships or depend on specific parameters. In soft groups, the group operation is defined softly under a given parameter, allowing for handling sets whose elements can change depending on the situation or specific conditions.

The primary motivation for the soft groupoid is to address situations where the elements of a system only interact partially or where only certain elements can be operated upon together under specific parameters. For example, in applications such as fuzzy data management or information systems where data is incomplete, or operations are only valid for particular subsets of the data, soft groupoids provide a more flexible and adaptive tool to accommodate such situations.

In this paper, we examine soft groupoids under the intersection operation (AND) and the union operation, as these aspects have not been discussed in detail in previous works (Ghosh *et al*., 2016; Oguz, G., Icen, I. ve Gürsoy, 2020; Oguz, 2023; Voigt, 2022). Furthermore, we will investigate the properties of soft subgroupoids and the collection of soft subgroupoids of a given soft groupoid under the intersection and union operations.

## RESEARCH METHODOLOGY

This research will begin with a literature review on soft groups, soft groupoids, and soft sub-groupoids, followed by the formal definition of relevant concepts and notation. In the following, a mathematical model for soft groupoids under the operations of intersection, AND, and union, as well as the properties of soft sub-groupoids, will be theoretically analyzed. Subsequently, the research will present several examples and cases to illustrate the theoretical results, followed by a discussion and comparison with previous literature to examine how these findings complement or extend existing research. Finally, conclusions will be drawn based on the analysis, highlighting the implications and potential for further study.

In this section, we present several definitions and related properties that will be used in the discussion section. Additionally, we use the symbols for intersection $\bigcap$, union $\bigcup$, and subset $\subseteq$ for standard sets, whereas distinct symbols for intersection $\sqcap$, union $\sqcup$, and subset $\sqsubseteq$ are used for soft sets.

**Definition 2.1** (Alajlan & Alghamdi, 2023; Barzegar *et al*., 2023) Let $\sigma$ be a function from the parameter set $A$ to the power set $P(S)$. A soft set over $S$ is presented as

$$\sigma_S \stackrel{\text{def}}{=} \{(a, \sigma(a)) \mid a \in A, \sigma(a) \subseteq S\}.$$

**Definition 2.2** Let $\sigma_A$ and $\rho_B$ be soft set over $S$. We say that $\sigma_A$ is a soft subset of $\rho_B$, denoted by $\sigma_A \sqsubseteq \rho_B$, if $A \subseteq B$ and $\sigma(a) \subseteq \rho(a)$ for each $a \in A$. The soft set $\sigma_A$ and $\rho_B$ are said to be soft equal if $\sigma_A \sqsubseteq \rho_B$ and $\rho_B \sqsubseteq \sigma_A$.

**Definition 2.3** Let $\sigma_A$ and $\rho_B$ be soft set over $S$ such that $A \bigcap B \neq \emptyset$. The restricted intersection of $\sigma_A$ and $\rho_B$, denoted by $\sigma_A \sqcap \rho_B$, forms the soft set $\xi_C = \{(c, \xi(c)) | c \in C, T: C \longrightarrow P(S)\}$, where $C = A \bigcap B$ and $\xi(c) = \sigma(c) \bigcap \rho(c)$ for every $c \in C$.

**Definition 2.4** If $\sigma_A$ and $\rho_B$ are two soft sets over $S$, then the AND operation between $\sigma_A$ and $\rho_B$, denoted by $\sigma_A \wedge \rho_B$, is defined as $\kappa_{A \times B}$, where for every $(a, z) \in A \times B$, $\kappa_{A \times B}(a, z) = \sigma(a) \bigcap \rho(z)$.

**Definition 2.5** Let $\sigma_A$ and $\rho_B$ be soft set over $S$. The union of $\sigma_A$ and $\rho_B$, denoted as $\sigma_A \sqcup \rho_B$, forms the soft set $\xi_C = \{(c, \xi(c)) | c \in C, T: C \longrightarrow P(S)\}$, where $C = A \cup B$, and function $\xi$ is defined as follows:

$$\xi(c) = \begin{cases} \sigma(c), & c \in A - B \\ \rho(c), & c \in B - A \\ \sigma(c) \cap \rho(c), & c \in A \cap B \end{cases}$$

for every $c \in C$.

**Definition 2.6** (Kandasamy, 2003) A nonempty set of elements $S$ is considered to form a groupoid if the binary operation exists on $S$, represented by $*$, such that for any elements $a$ and $z$ in $S$, the product $a * z$ is also in $S$.

**Definition 2.7** (Kandasamy, 2003) Suppose $S$ is a groupoid. A proper subset $R \subset S$ is a subgroupoid and is denoted by $R < S$, if $R$ is itself a subgroupoid

## RESULTS AND DISCUSSION

Before proceeding to the discussion, we present the definition of a soft groupoid, which we have derived from the work of (Acar *et al*., 2010; Aktaş & Çağman, 2007; Alajlan & Alghamdi, 2023; Barzegar *et al*., 2023; Çelik *et al*., 2011; Feng *et al*., 2008), as a foundation for moving forward to other sections.

**Definition 4.1** Let $\sigma_B$ be a soft set over groupoid $S$. If for each $d \in B$, $\sigma(d)$ is a subgroupoid of $S$, then $\sigma_B$ It is called a soft groupoid over $S$.

As an illustration of Definition 4.1, we provide an example involving a set of $2 \times 2$ matrices over the integers equipped with a specific binary operation.

**Example 4.2.** Let $S = \left\{ \begin{pmatrix} a & x \\ z & d \end{pmatrix} \middle| a, x, z, d \in \mathbb{Z} \right\}$ be a groupoid under the matrix subtraction operation, $B = 2\mathbb{Z}$ and $X = 8\mathbb{Z}$. Consider the functions $\sigma: B \longrightarrow P(\mathbb{Z})$ and $\rho: X \longrightarrow P(\mathbb{Z})$ defined by

$$\sigma(d) = \left\{ \begin{pmatrix} dc & dc \\ 0 & 0 \end{pmatrix} \middle| c \in \mathbb{Z} \right\} \text{ and } \rho(x) = \left\{ \begin{pmatrix} xc & xc \\ 0 & 0 \end{pmatrix} \middle| c \in \mathbb{Z} \right\}$$

for any $d \in B$ and $x \in X$. Thus, we can conclude that $\sigma(a)$ and $\rho(x)$ Are subgroupoid of $S$. Therefore, we obtain that, $\sigma_A$ and $\rho_X$ are soft groupoids over $S$.

Before proceeding to the next property of soft groupoids, we present the property of the intersection of two or more subgroupoids of a groupoid. This property plays an essential role in the properties we will examine.

**Proposition 4.3** Let $S$ be a groupoid and $\Lambda$ be an index set. Suppose $\mathcal{L} = \{H_\alpha | \alpha \in \Lambda\}$, where $H_\alpha$ is a subgroupoid of $S$ for every $\alpha \in \Lambda$. The intersection of all the subgroupoids in $\mathcal{L}$, i.e., $\bigcap_{\alpha \in \Lambda} H_\alpha \neq \emptyset$ is a subgroupoid of $S$.

**Proof.** Since $H_\alpha$ is a subgroupoid of $S$ for every $\alpha \in \Lambda$, we have $H_\alpha \subseteq S$ and so $\bigcap_{\alpha \in \Lambda} H_\alpha \subseteq H_\alpha \subseteq S$. Furthermore, let $a, z \in \bigcap_{\alpha \in \Lambda} H_\alpha$, we have $a, z \in H_\alpha$ for every

$\alpha \in \Lambda$. Because of $H_\alpha$ is a subgroupoid of $S$ for every $\alpha \in \Lambda$. Thus, $az \in H_\alpha$ and so $az \in \bigcap_{\alpha\in\Lambda} H_\alpha$. In other words, $\bigcap_{\alpha\in\Lambda} H_\alpha \neq \emptyset$ is a subgroupoid of $S$.

**Corollary 4.4** The nonempty intersection of two subgroupoids of a groupoid is itself a subgroupoid.

In the following section, we present the properties associated with the intersection operation, AND, and the union of soft groupoids, where Corollary 4.4 is particularly useful in the proof process.

**Proposition 4.5** The intersection of two soft groupoids $\sigma_A$ and $\rho_A$ over a groupoid $S$, denoted by $\sigma_A \sqcap \rho_A$, is always a soft groupoid.

**Proof.** Let $\sigma_A$ and $\rho_A$ be soft groupoid over groupoid $S$. According to Definition 4.1, $\sigma(a)$ and $\rho(a)$ are subgroupoids of $S$, for any $a \in A$. Therefore, based on Corollary 4.4, $\sigma(a)\bigcap\rho(a)$ is a subgroupoid of $S$. As a result, following Definition 4.1, $\sigma_A \sqcap \rho_A$ is a soft groupoid over $S$. ■

**Proposition 4.6** The union of two soft groupoids $\sigma_A$ and $\rho_B$ over a groupoid $S$ where $A\bigcap B = \emptyset$, denoted by $\sigma_A \sqcup \rho_B$, is always a soft groupoid.

**Proof.** Let $\sigma_A$ and $\rho_B$ be soft groupoids over groupoid $S$. Suppose $\sigma_A \sqcup \rho_A = \eta_C$ where $C = A\bigcup B$. Because $A\bigcap B = \emptyset$, it implies that either $c \in A - B$ or $c \in B - A$ for any $c \in C$. If $c \in A - B$, then $\eta(c) = \sigma(c)$ is a subgroupoid of $S$, and if $c \in B - A$, then $\eta(c) = \rho(c)$ is a subgroupoid of $S$. As a result, $\sigma_A \sqcup \rho_B$ is a soft groupoid over $S$.■

**Proposition 4.7** Let $\sigma_A$ and $\rho_B$ be soft groupoids over groupoid $S$. Then $\sigma_A \wedge \rho_B$, it is a soft groupoid over $S$.

**Proof.** Suppose $\sigma_A$ and $\rho_B$ be soft groupoids over groupoid $S$. By Definition 4.1, $\sigma(a)$ and $\rho(z)$ are subgroupoids of $S$ for any $a \in A$ and $z \in B$. Based on Definition 2.4, it can be written as $\sigma_A \wedge \rho_B = \kappa_{A\times B}$. Since $\sigma(a)$ and $\rho(z)$ are subgroupoids of $S$ for any $a \in A$ and $z \in B$, based on Corollary 4.4, we have that $\kappa(a,z) = \sigma(a)\bigcap\rho(z)$ is a subgroupoid of $S$. In other words, $\kappa(a,z)$ is a subgroupoid of $S$ for any $(a,z) \in A \times B$. Thus, $\sigma_A \wedge \rho_B$ is a soft groupoid over $S$. ■

In the previous section on properties, we examined the intersection operation, AND, and the union in soft groupoids. Afterward, we will study soft subgroupoids related to the collection of soft subgroupoids associated with the intersection, AND, and union operations.

**Definition 4.8** Let $\sigma_B$ and $\rho_X$ be soft groupoid over groupoid $S$. The soft groupoid $\rho_X$ is considered a soft subgroupoid of $\sigma_B$, denoted as $\rho_X \lesseqqgtr \sigma_B$ if the following conditions are satisfied:

1. $X$ is a subset of $B$
2. $\rho(z) < \sigma(z)$ for any element $z \in X$.

**Example 4.9.** Based on the conditions in Example 4.2, we have $X \subseteq B$ and $\rho(x) < \sigma(x)$ for any $x \in X$. Therefore, $\rho_X \mathrel{\widetilde{\leqq}} \sigma_B$.

**Remarks 4.10** Let $\sigma_X$ and $\rho_X$ be two soft groupoid over groupoid $S$. If $\rho(x) < \sigma(x)$ for any $x \in X$, then $\rho_X \mathrel{\widetilde{\leqq}} \sigma_X$.

**Proof**. A straightforward implication of Definition 4.8. ∎

**Remarks 4.11** If $\sigma_S$ is a soft groupoid over groupoid $S$, then $\sigma_S \mathrel{\widetilde{\leqq}} \sigma_S$

**Proof**. A natural outcome of Definition 4.8. ∎

**Proposition 4.12** Suppose $\sigma_B$ is a soft groupoid over the groupoid $S$, and $\left\{\rho_{i_{X_i}} \middle| i \in I\right\}$ represents a nonempty collection of soft subgroupoids of $\sigma_B$, where I denote an index set. Then

1. $\sqcap_{i\in I} \rho_{i_{X_i}}$is a soft subgroupoid of $\sigma_B$.
2. $\wedge_{i\in I} \rho_{i_{X_i}}$ is a soft subgroupoid of $\sigma_B$.
3. If $X_i \cap X_j = \emptyset$ for any $i \neq j$, then $\sqcup_{i\in I} \rho_{i_{X_i}}$ is a soft subgroupoid of $\sigma_B$.

**Proof**.

1. Suppose $\left\{\rho_{i_{X_i}} \middle| i \in I\right\}$ is a collection of soft subgroupoids of $\sigma_B$. In that case $\cap_{i\in I} X_i \subseteq B$. Because of $\rho_{i_{X_i}} \subseteq \sigma_B$ for any $i \in I$ and $\rho_i(x) < \sigma(x)$ for any $x \in X_i$. As a result, by Proposition 4.3, we have $\cap_{i\in I} \rho_i(x) < \sigma(x)$ for any $x \in X_i$. Therefore, by Definition 4.8, we conclude, $\sqcap_{i\in I} \rho_{i_{X_i}}$ is a soft subgroupoid of $\sigma_B$.
2. In the following step, $\rho_i(x) < \sigma(x)$ for any $x \in X_i$ where $i \in I$. Consequently, based on Proposition 4.3, we have $\cap_{i\in I} \rho_i(x) < \sigma(x)$ for any $x \in X_i$. Therefore, by Definition 4.8, we conclude, $\wedge_{i\in I} \rho_{i_{X_i}}$ is a soft subgroupoid of $\sigma_B$.
3. Finally, let $\sqcup_{i\in I} \rho_{i_{X_i}} = \xi_C$ where $X_i \cap X_j = \emptyset$ for any $i \neq j$. Thus, based on Definition 2.6, we have $C = \cup_{i\in I} X_i$ and $\xi(x) = \cup_{i\in I} \rho_i(x)$. The consequence of $\left\{\rho_{i_{X_i}} \middle| i \in I\right\}$ being a collection of soft subgroupoids of $\sigma_B$ is that we have $\rho_i(x) < \sigma(x)$ For any $x \in X_i$ where $i \in I$. Since $X_i \cap X_j = \emptyset$ for any $i \neq j$. Therefore, for any $x \in C$, there exists a unique $i_a \in I$ such that $\xi(x) = \rho_{i_a}(x)$. Thus, $\xi(x) = \cup_{i\in I} \rho_i(x) < \sigma(x)$. Hence, based on Definition 2.6, we obtain $\sqcup_{i\in I} \rho_{i_{X_i}}$ is a soft subgroupoid of $\sigma_B$. ∎

## CONCLUSION

Based on the results and discussion of this paper, we have that the intersection, AND, and union operations on two soft groupoids result in a soft groupoid. Furthermore, the intersection, AND, and union operations on a collection of soft subgroupoids result in a soft subgroupoid.

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