



Students' Mathematical Reasoning: A Systematic Literature Review with a Focus on Metaphorical Thinking-Based Learning Approaches

Annisa Desti Fahjri, Siti Fatimah*, and Jarnawi Afgani Dahlan

Mathematics Education Study Program, Universitas Pendidikan Indonesia, Bandung, Indonesia

*sitifatimah@upi.edu

Article Info	Abstract
Article history: Received: April 22, 2026 Accepted: May 30, 2026 Published: June 13, 2026 Keywords: Conceptual metaphors Instructional models Mathematics education Mathematical reasoning Metaphorical thinking This article is licensed under a Creative Commons Attribution-ShareAlike 4.0 International License. © 2026 Journal of Mathematics, Science, and Computer Education	Mathematical reasoning is a fundamental component of meaningful mathematics learning; however, research on instructional approaches that systematically support its development remains fragmented. In particular, studies on mathematical reasoning and metaphorical thinking have largely evolved independently, resulting in limited integrative insights. Studies on mathematical reasoning and metaphorical thinking have often developed separately, resulting in limited understanding of how metaphorical-thinking-based learning contributes to reasoning development. This systematic literature review (SLR) aims to synthesize research trends on students' mathematical reasoning in mathematics education and to examine how metaphorical-thinking-based learning approaches support the development of mathematical reasoning. Following the PRISMA 2020 framework, a systematic search of the Scopus and ERIC databases identified 36 peer-reviewed journal articles published between 2017 and 2025. The selected studies were analyzed using reflexive thematic analysis focusing on research trends, instructional implementations, metaphor types, learning models, and reported effects on students' mathematical reasoning. The findings indicate that mathematical reasoning is increasingly viewed as a multidimensional construct involving cognitive, representational, social, and contextual aspects. The review also reveals that metaphorical thinking functions as a cognitive bridge that connects abstract mathematical concepts with students' concrete experiences through conceptual, structural, orientational, cultural, and embodied metaphors. These metaphorical approaches are integrated into various instructional models, including Realistic Mathematics Education, ethnomathematics, problem-based learning, embodied learning, and technology-supported learning environments. Furthermore, the findings highlight several research gaps, including limited empirical studies explicitly linking metaphorical thinking and mathematical reasoning, insufficient longitudinal evidence, and limited comparative analysis across metaphor types. Overall, this review emphasizes the need for more theoretically integrated and empirically grounded studies on metaphor-based mathematical reasoning in mathematics education.
To cite this article: Fahjri, A. D., Fatimah, S., & Dahlan, J. A. (2026). Students' mathematical reasoning: A systematic literature review with a focus on metaphorical thinking-based learning approaches. <i>Journal of Mathematics, Science, and Computer Education</i>, 6(1), 387-402.	

INTRODUCTION

Mathematical reasoning remains a major challenge in mathematics education worldwide. Numerous studies have reported that students often experience difficulties in constructing mathematical arguments, making generalizations, drawing logical inferences, and justifying solution strategies. These difficulties are reflected in large-scale international assessments, which consistently show that many students struggle with higher-order mathematical tasks requiring reasoning, modeling, and problem-solving. As a result, improving students' mathematical reasoning has become an important priority for mathematics education research and practice.

In the Indonesian context, the need to strengthen mathematical reasoning is underscored by international assessment results. Reports from the Program for International Student Assessment (PISA) indicate that Indonesian students' mathematical literacy remains below the OECD average, particularly on tasks requiring higher-order reasoning, modeling, and non-routine problem solving. These findings suggest persistent difficulties in generalization, inference, and mathematical justification, which align with national studies reporting challenges in argument construction, strategy explanation, and representation coordination (Hansen & Naalsund, 2025; Siregar et al., 2025).

Over the past decade, numerous studies have explored instructional strategies to enhance students' mathematical reasoning, including reasoning-oriented task design (Cervantes-Barraza & Araujo, 2023), technology- and web-based learning environments (Adelia et al., 2025), and pedagogical models emphasizing exploration, discussion, and active knowledge construction. Collectively, these studies indicate that reasoning develops most effectively when students are encouraged to formulate ideas, test conjectures, and reflect on their thinking.

Parallel to this line of research, metaphorical thinking has gained increasing attention as an approach to support students' understanding of abstract mathematical concepts. Metaphors allow learners to map abstract ideas onto concrete experiences, thereby enhancing conceptual understanding, problem-solving, and mental (Purwasi & Fitriyana, 2021; Scheiner et al., 2022). Research shows that metaphors operate not only through language but also through gestures, visualizations, and embodied cognition (Alcaraz-Carrión et al., 2022; Pantsar, 2018), and can support conceptual mapping and meaning-making in mathematics (Purwasi & Fitriyana, 2021; Setiani et al., 2018).

Despite the growth of research on both mathematical reasoning and metaphorical thinking, these two strands have largely developed independently. Previous reviews tend to focus on metaphors in relation to conceptual understanding or general literacy rather than mathematical reasoning (Falani et al., 2023; Ghifari, 2025), while studies on reasoning rarely examine metaphors as cognitive tools for inference, argumentation, or conceptual structuring. Moreover, limited research has systematically examined different types of metaphors (conceptual, structural, orientational, and gestural), their relative effectiveness, or the instructional models that integrate metaphorical thinking to support reasoning. To date, no Systematic Literature Review (SLR) has comprehensively integrated research trends on mathematical reasoning with metaphorical thinking in mathematics education. The novelty of this review lies in its integrative perspective. Unlike previous reviews that examined mathematical reasoning and metaphorical thinking as separate research domains (Falani et al., 2023; Ghifari, 2025), this study systematically synthesizes evidence on how metaphor-thinking-based learning approaches contribute to the development of students' mathematical reasoning. Furthermore, this review identifies dominant metaphor types, examines their instructional implementation across different learning models, and highlights research gaps that may inform future theoretical and empirical developments in mathematics education.

To address these gaps, this study conducts a SLR focusing on the relationship between metaphorical thinking and mathematical reasoning in mathematics education. The review aims to synthesize how metaphors are implemented, how they support reasoning development, the dominant types of metaphors employed, and the instructional models used. Accordingly, this study seeks to answer the following research questions:

- RQ1. What are the research trends on students' mathematical reasoning in mathematics education over the past decade?
- RQ2. How have metaphorical thinking-based learning approaches been implemented in mathematics education?
- RQ3. How does a metaphorical-thinking-based approach influence or support the development of students' mathematical reasoning?
- RQ4. What types of metaphors are commonly employed within metaphorical thinking-based learning approaches in mathematics education?
- RQ5. What learning models can be integrated with metaphorical thinking-based approaches in mathematics education, and how do they support students' mathematical reasoning?
- RQ6. What research gaps exist in studies examining the relationship between metaphorical thinking-based learning and students' mathematical reasoning?

Overall, this SLR is expected to contribute theoretically and practically to the development of metaphor-based mathematics learning designs and to enrich the literature on students' mathematical reasoning.

METHODS

Research Design

This study employed a SLR design to investigate the relationship between metaphorical thinking and students' mathematical reasoning in mathematics education. Systematic literature reviews are widely used to synthesize evidence from existing studies, identify research trends, and reveal conceptual gaps within a particular field of inquiry ([Connett, 2018](#)). Accordingly, the SLR approach was selected to systematically examine previous studies and identify research trends, conceptual relationships, and gaps concerning metaphorical thinking and mathematical reasoning. The review process followed the PRISMA 2020 framework, which consists of four stages: identification, screening, eligibility assessment, and inclusion of studies. The literature review process was conducted in accordance with the PRISMA guidelines to ensure methodological rigor and transparency ([McKenzie et al., 2017](#)).

Subject of Research

The population of this study consisted of journal articles related to mathematical reasoning and metaphorical thinking in mathematics education. Data collection involved a systematic search of two major databases, Scopus and ERIC. Keywords and search strings were developed through an exploratory theoretical search, incorporating core terms, related concepts, and synonyms to maximize coverage, as recommended by [Falani et al. \(2023\)](#). The search terms included "mathematical reasoning," "reasoning ability," "mathematical thinking," "metaphorical thinking," "conceptual metaphor," and "mathematics education," supplemented by database-suggested terms and online thesauri. In line with previous studies, the development of keywords also utilized online thesauri, terminology from prior research, and automatically suggested keywords from databases such as Scopus and ERIC.

The literature search was conducted between November and December 2026, with the final search completed on February 30, 2026. In addition to the database search, a limited manual snowballing procedure was conducted by reviewing the reference lists of key articles

on metaphorical thinking and mathematical reasoning to identify potentially relevant studies not captured by the initial search.

The publication period was limited to studies published between 2017 and 2025 to capture recent developments in mathematics education research, particularly the growing emphasis on reasoning-oriented learning, technology-supported instruction, and metaphorical thinking approaches. This time period was also selected to ensure the inclusion of contemporary pedagogical trends and theoretical developments relevant to current mathematics learning contexts.

Using the Scopus and ERIC databases, the researchers conducted a literature search by applying the predetermined keywords and constructing a comprehensive search string. In Scopus, the search was performed using the TITLE-ABS-KEY field with the following search string: ("reasoning" OR "reasoning ability" OR "reasoning skill" OR "metaphorical thinking") AND ("mathematics education" OR "mathematics learning" OR "mathematics teaching" OR "learning mathematics").

Meanwhile, in ERIC, the search was conducted through the descriptor and abstract fields using the following search string: "metaphorical thinking" AND "mathematics education." The selection of these two databases was based on recommendations from previous systematic studies, which emphasize that using more than one database is necessary to broaden the search coverage and reduce selection bias. [Braun & Clarke \(2022\)](#) note that multiple databases enhance the comprehensiveness of the search process. Scopus was selected because it is a globally recognized scientific citation database widely used in SLRs and meta-analyses, while ERIC is considered highly relevant for literature related to education and mathematics learning. From the initial search conducted across both databases, a total of 2,789 articles were obtained before entering the screening stage.

Instruments

The instruments used in this study consisted of inclusion–exclusion criteria and a methodological quality appraisal checklist. The inclusion and exclusion criteria were used to ensure that only studies relevant to the research objectives were retained during the screening process, as presented in Table 1.

Table 1 Inclusion and exclusion

Criteria	Inclusion	Exclusion
Title and content	Meeting research requirements	Does not meet research requirements
Study limitations	Limited to studies published from 2017 to 2025	Studies published before 2017
Publication type	Journal articles	Conference proceedings, chapters in books, book series, books, etc.
Article field of study	Research in mathematics learning	Others
Language	English	Others
Access	Open access	Preview article or paid article

In addition, a methodological quality assessment was conducted on the final 36 selected articles using the critical appraisal framework proposed by [Connett \(2018\)](#). A checklist was applied to evaluate methodological rigor, focusing on: (1) Relevance of research questions, (2) Clarity of study aims, (3) Appropriateness of research design, (4) Ethical considerations, (5) Rigor of data analysis, (6) Clarity of findings (7) Overall contribution of the study.

The article selection process was conducted in two stages: title–abstract screening and full-text screening. During the first stage, all retrieved articles were reviewed based on the

predefined inclusion and exclusion criteria, including publication year, publication type, language, accessibility, and relevance to mathematical reasoning, metaphorical thinking, and mathematics education. Articles that did not meet these criteria were excluded.

In the second stage, the full texts of the remaining articles were examined in greater depth to determine their relevance to the objectives of this review. Particular attention was given to whether the studies explicitly addressed mathematical reasoning, metaphorical thinking, or the relationship between these constructs within mathematics education contexts. No duplicate records were identified across the Scopus and ERIC databases. To ensure consistency throughout the selection process, all articles were evaluated using the same eligibility criteria, and screening decisions were documented at each stage in accordance with the PRISMA framework. Only studies meeting acceptable quality standards were included in the final synthesis.

Data Analysis Technique

The 36 articles that passed the quality assessment were qualitatively analyzed using a reflexive thematic analysis approach to identify patterns, themes, and relationships across the findings. The analysis began with a comprehensive reading of each article, including the abstract, methodology, results, and discussion sections, to develop an in-depth understanding of the research context and focus. At this stage, all findings relevant to the research questions—particularly those related to mathematical reasoning and metaphorical thinking in mathematics education—were systematically extracted and summarized.

The next stage involved the initial coding process, which entailed identifying and labeling key units of information from each article. In accordance with the guidelines for reflexive thematic analysis proposed by [Braun & Clarke \(2022\)](#), the coding process was conducted in a flexible and iterative manner, involving the researcher's active interpretation of the data. The initial codes were then clustered based on recurring patterns or concepts identified across the included studies.

The generated codes were subsequently examined through a constant comparison process to ensure consistency and analytical depth. At this stage, findings across articles were compared to identify similarities, variations, contradictions, and potential conceptual relationships. This procedure aligns with the recommendations of [Nowell et al. \(2017\)](#), who emphasize the importance of iterative evaluation to enhance the credibility of thematic analysis results.

In developing the final themes and subthemes, the researchers synthesized meanings across diverse research contexts using a qualitative synthesis approach as described by [Braun & Clarke \(2022\)](#). This synthesis process involved integrating similar findings, exploring relationships among research variables (e.g., the relationship between metaphorical representations and the development of students' mathematical reasoning), and identifying the distinctive contributions of each article to construct a more comprehensive understanding.

After the final themes were established, the researchers constructed a synthesis narrative that comprehensively presented research trends, key patterns of findings, remaining research gaps, and the conceptual contributions of each theme. This narrative subsequently formed the basis for the findings and discussion sections of this SLR.

Research Procedures

The research procedures in this study followed the PRISMA 2020 framework and consisted of four stages, namely identification, screening, eligibility assessment, and inclusion, as illustrated in Figure 1.

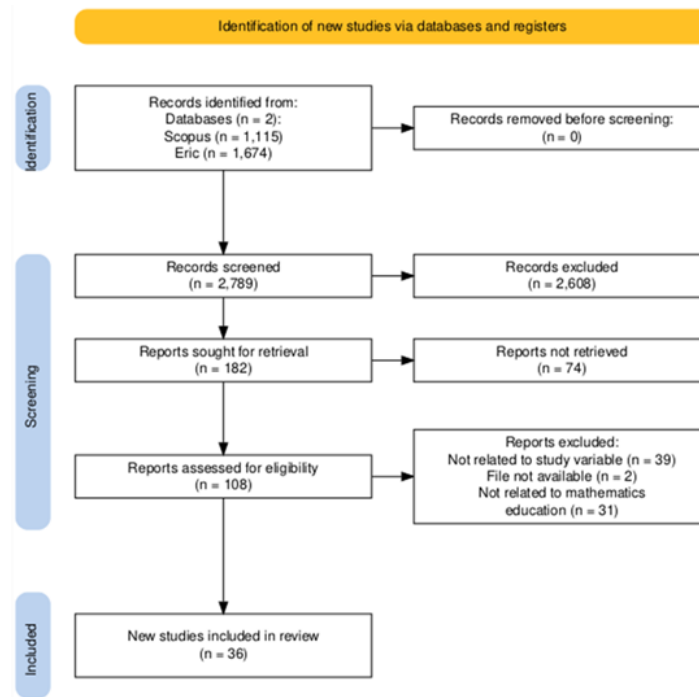


Figure 1 Articles screening process

Stage 1: Title–Abstract Screening

The identification stage yielded a total of 2,789 records from two databases, consisting of 1,115 articles from Scopus and 1,674 articles from ERIC. No duplicate records were identified during the initial review process. Therefore, all retrieved articles proceeded to the screening stage.

Stage 2: Full-Text Screening

During the screening stage, the titles and abstracts of the 2,789 articles were examined using the predefined inclusion and exclusion criteria. A total of 2,608 articles were excluded because they did not meet the scope of the review, leaving 181 articles for further consideration. Full-text retrieval was then attempted for these articles; however, 74 articles could not be accessed, resulting in 108 articles being assessed during the eligibility stage.

At the eligibility stage, the full texts were reviewed in detail. Seventy-two articles were excluded because they were not directly related to the variables under investigation ($n = 39$), the full text was unavailable ($n = 2$), or the studies were not situated within mathematics education contexts ($n = 31$). Consequently, 36 articles met all eligibility criteria and were included in the final review. This rigorous screening process aligns with recommendations by Ghifari (2025), who emphasizes careful filtering due to conceptual overlap in metaphor-related research.

RESULTS AND DISCUSSION

RQ1. What are the research trends on students' mathematical reasoning in mathematics education over the past decade?

The reviewed studies indicate that research on students' mathematical reasoning has developed in increasingly diverse directions over the past decade. Mathematical reasoning is examined in relation to proof, conceptual understanding, problem solving, generalization, and mathematical representation (Jeannotte & Kieran, 2017; Schindler et al., 2017). The growth of publications in this field is illustrated in Figure 2.

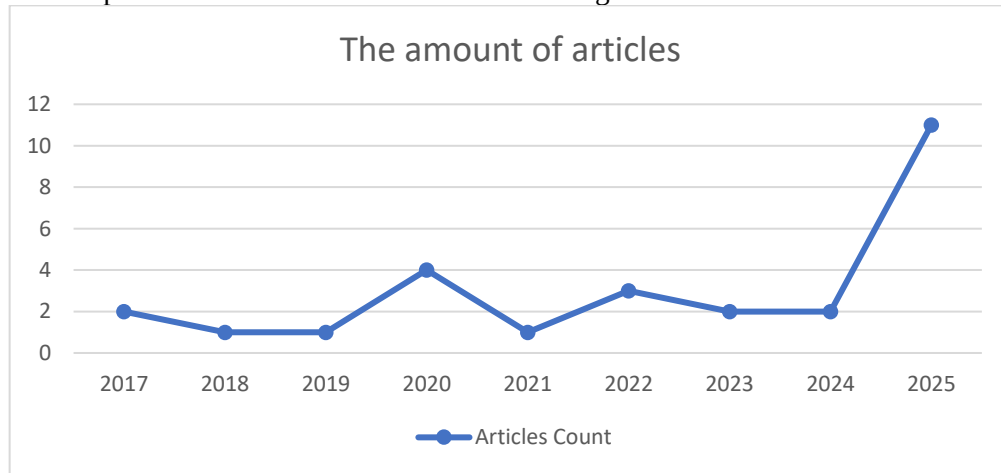


Figure 2 Publication trends in mathematical reasoning research (2017-2025)

Figure 2 shows that the number of publications on mathematical reasoning has increased over time, with a noticeable rise after 2020 and a peak in recent years. Several studies report the incorporation of authentic and contextual learning environments in examining mathematical reasoning. For instance, Fraihat et al., (2022) investigate reasoning within situated learning contexts, while Cervantes-Barraza and Cervantes-Barraza & Araujo (2023) examine the role of cultural contexts in supporting students' reasoning processes. The geographical distribution of studies is presented in Figure 3.

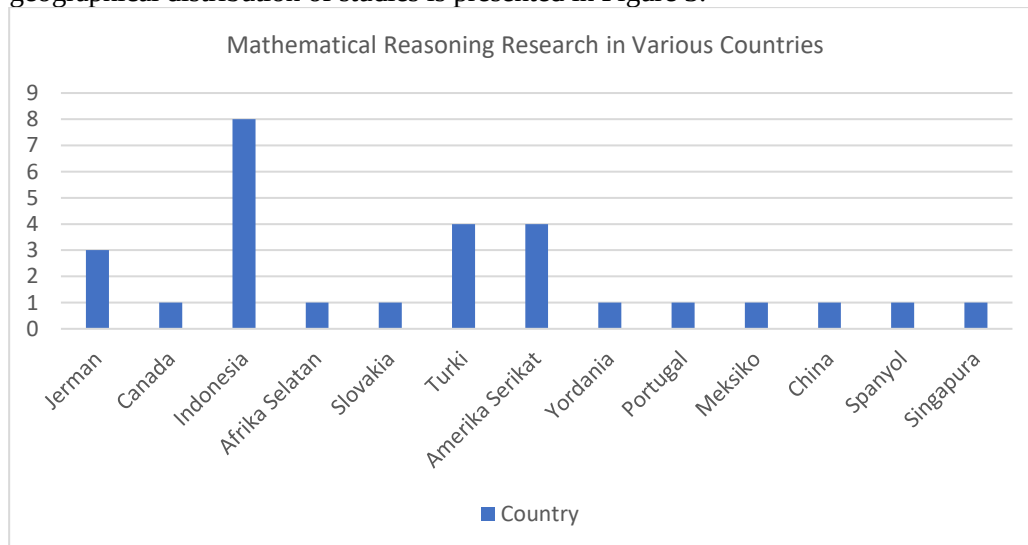


Figure 3 Geographical distribution of studies on mathematical reasoning in mathematics education

Figure 3 indicates that contributions come from various countries, with Indonesia emerging as one of the major contributors, followed by other regions such as Turkey, the United States, and European countries. A number of studies also focus on mathematical reasoning in collaborative settings. Hansen & Naalsund (2025) analyze interaction patterns in collaborative reasoning and describe how students engage in shared reasoning processes during mathematical activities. The relationships among research themes are visualized through keyword co-occurrence networks, as shown in Figure 4.

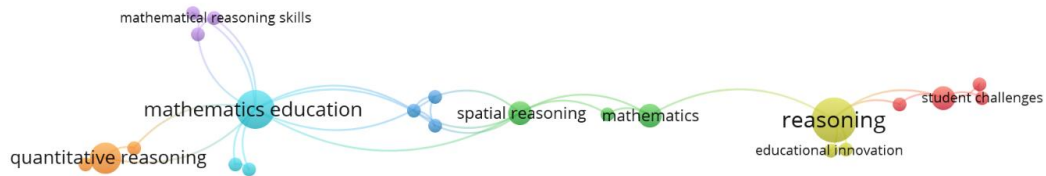


Figure 4 Network visualization of Research about mathematical reasoning

Figure 4 illustrates the connections among key terms in the literature. The term “reasoning” appears as a central node, connected to themes such as mathematics education, educational innovation, and student challenges. The temporal development of research topics is further illustrated in Figure 5.

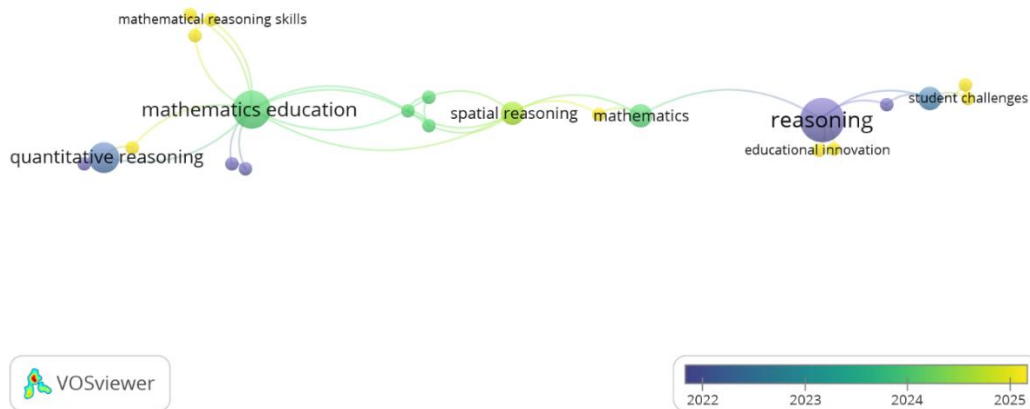


Figure 5 Overlay visualization of Research about mathematical reasoning

Figure 5 shows the evolution of research topics over time, where earlier studies emphasize topics such as “quantitative reasoning,” while more recent studies highlight themes related to educational innovation and student challenges.

In terms of specific domains, the literature identifies spatial reasoning, covariational reasoning, and representation-based reasoning as key areas of focus. Studies by Dhlamini et al., (2019) and Hasanah et al., (2024) examine spatial reasoning in relation to geometry and cognitive development, while Kertil (2020) investigates covariational reasoning in dynamic representations.

The relationship between mathematical reasoning and conceptual understanding is also frequently reported. Bakar et al., (2018) and Schindler et al., (2017) document associations between students’ reasoning and conceptual misconceptions. Hamidah et al., (2025) report variations in reasoning ability among preservice teachers.

Methodologically, the reviewed studies employ a range of approaches, including mixed methods, case studies, design-based research, and lesson study. Martins et al., (2023)

examine the use of lesson study in supporting the development of instructional practices related to reasoning.

RQ2. How have metaphorical thinking-based learning approaches been implemented in mathematics education?

The reviewed studies show that metaphorical-thinking-based approaches have been implemented through various instructional practices and learning environments in mathematics education. Most studies describe the use of metaphorical thinking as part of classroom activities designed to connect students' prior experiences with mathematical concepts.

Several studies implemented metaphorical thinking through analogy-based mathematical tasks and contextual activities. [Purwasi & Fitriyana \(2021\)](#), for example, used visual and verbal analogies to support students' reasoning and problem-solving processes. Similarly, [Setiani et al., \(2018\)](#) integrated metaphorical thinking into Model-Eliciting Activities (MEA) to support mathematical literacy and self-efficacy.

Other studies reported the implementation of metaphorical thinking through teacher explanations and classroom discourse. [Hendriana et al. \(2017\)](#) examined how metaphorical thinking learning supported mathematical questioning activities, while [Scheiner et al., \(2022\)](#) described the use of metaphors in mathematics teaching practices and instructional preparation.

Metaphorical approaches were also implemented in technology-supported learning environments. [Ghifari \(2025\)](#) identified the integration of metaphors into digital learning designs, including virtual manipulations and visual representations. In addition, [Adelia et al., \(2025\)](#) reported the development of web-based learning environments that supported reasoning processes through interactive representations.

Furthermore, several studies implemented metaphorical thinking within contextual and cultural learning settings. [Cervantes-Barraza & Araujo \(2023\)](#) designed ethnomathematical tasks that connected mathematical reasoning with cultural experiences and authentic situations.

Overall, the reviewed studies indicate that metaphorical-thinking-based approaches have been implemented through analogy-based tasks, classroom discourse, contextual activities, technology-supported environments, and culturally situated mathematical learning.

RQ3. How does a metaphorical-thinking-based approach influence or support the development of students' mathematical reasoning?

The reviewed studies identify several mechanisms through which metaphorical thinking is associated with students' mathematical reasoning. Table 2 summarizes the mechanisms and the corresponding studies.

The reviewed studies indicated that metaphorical thinking supports students' mathematical reasoning through several mechanisms, including cognitive bridging between abstract and concrete concepts, the development of visual and gestural mental representations, increased cognitive engagement in mathematical activities, contextual reasoning through cultural experiences, and technology-supported interactive learning environments ([Adelia et al., 2025](#); [Alcaraz-Carrión et al., 2022](#); [Cervantes-Barraza & Araujo, 2023](#); [Purwasi & Fitriyana, 2021](#)).

Table 2 Mechanisms supporting mathematical reasoning in metaphorical thinking-based learning

Mechanism	Description	Supporting Studies
Cognitive bridging	Connecting abstract mathematical concepts with concrete experiences	Purwasi & Fitriyana (2021; Scheiner et al., (2022)
Mental representation	Supporting visual, gestural, and symbolic representations	Alcaraz-Carrión et al., (2022; Hicks (2024)
Cognitive engagement	Encouraging analogy construction, pattern exploration, and argumentation	Hendriana et al., (2017; Setiani et al., (2018)
Contextual reasoning	Relating mathematical ideas to cultural and real-life contexts	Cervantes-Barraza & Araujo (2023)
Technology-supported reasoning	Using digital representations and dynamic visualization	Adelia et al., (2025; Kertil (2020)

RQ4. What types of metaphors are commonly employed within metaphorical thinking-based learning approaches in mathematics education?

The literature identifies several types of metaphors used in mathematics education:

1. Conceptual metaphors, which are used to explain abstract mathematical ideas through familiar concepts and experiences (Falani et al., 2023; Purwasi & Fitriyana, 2021).
2. Structural metaphors, which map relationships between mathematical structures and more familiar systems or patterns (Hicks, 2024).
3. Orientational metaphors, involving spatial representations such as directions, positions, and number-line orientations (Schindler et al., 2017).
4. Cultural or ethnomathematical metaphors, which connect mathematical concepts with cultural practices and local experiences (Cervantes-Barraza & Araujo, 2023).
5. Gestural or embodied metaphors, represented through bodily movements, gestures, and sensorimotor experiences in mathematical communication (Alcaraz-Carrión et al., 2022).

RQ5. What learning models can be integrated with metaphorical thinking-based approaches in mathematics education, and how do they support students' mathematical reasoning?

The reviewed studies indicate that metaphorical thinking is integrated with various instructional models. These include Realistic Mathematics Education (RME), ethnomathematics, embodied learning, problem-based learning, task-based learning, Conceptual PlayWorld, and technology-based learning models. The integration is reflected in the use of analogies, representations, contextual tasks, and interactive learning environments (Adelia et al., 2025; Alcaraz-Carrión et al., 2022; Cervantes-Barraza & Araujo, 2023; Kertil, 2020).

RQ6. What research gaps exist in studies examining the relationship between metaphorical thinking-based learning and students' mathematical reasoning?

The findings identify several research gaps related to the relationship between metaphorical thinking and mathematical reasoning. The relationship between these constructs is illustrated in Figure 6.

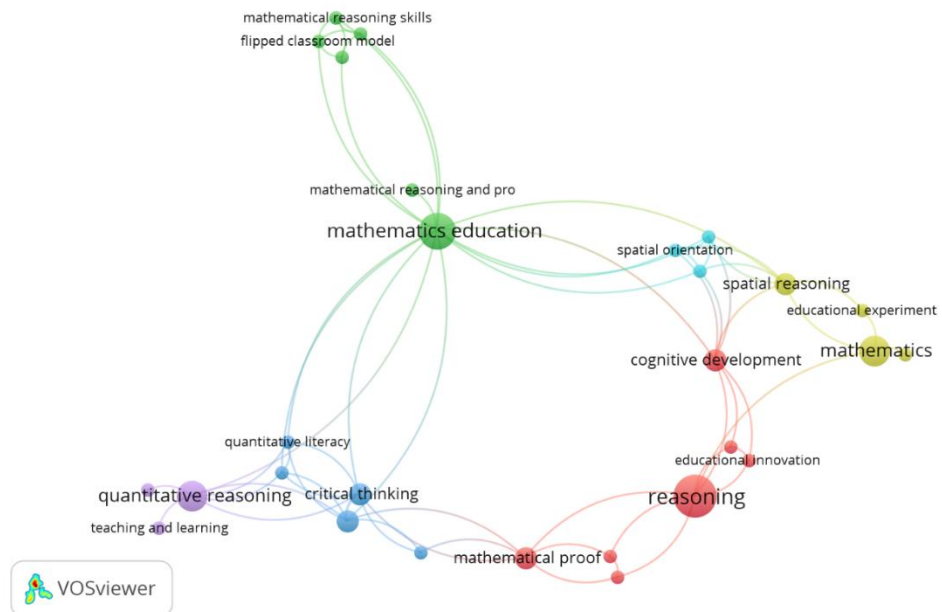


Figure 6 Network visualization of relationship between metaphorical thinking-based learning and students' mathematical reasoning

Figure 6 shows that metaphorical thinking does not appear as a dominant or strongly connected keyword within the broader network of mathematical reasoning research. The findings identify several research gaps: limited empirical studies explicitly linking metaphorical thinking and mathematical reasoning, limited exploration of different types and functions of metaphors, lack of longitudinal studies examining the development of reasoning over time, imbalance across educational levels, limited integration of interdisciplinary theoretical frameworks, and limited use of technology-based approaches in studying metaphorical thinking and reasoning.

The findings of this study reveal that research on mathematical reasoning and metaphorical thinking in mathematics education has developed in increasingly diverse yet fragmented directions. On the one hand, the literature demonstrates a clear expansion in how mathematical reasoning is conceptualized; on the other hand, the integration between reasoning and metaphorical thinking remains limited and insufficiently theorized.

From the perspective of research trends, mathematical reasoning is no longer confined to a purely cognitive domain but is increasingly examined within social, cultural, and contextual dimensions. Studies such as [Hansen & Naalsund \(2025\)](#) highlight the importance of interaction patterns in collaborative reasoning, while [Cervantes-Barraza & Araujo \(2023\)](#) demonstrate how cultural contexts enrich students' reasoning processes. Similarly, [Fraihat et al., \(2022\)](#) emphasize the role of situated learning in enhancing reasoning and proof. These findings collectively indicate a shift toward viewing reasoning as a socially mediated practice.

However, despite this expansion, a critical issue emerges regarding theoretical coherence. Many studies adopt contextual and collaborative approaches without explicitly grounding them in comprehensive theoretical frameworks. As a result, the field shows strong empirical growth but lacks conceptual consolidation. This limitation is further reflected in the inconsistent treatment of different types of reasoning. While domains such as spatial reasoning ([Dhlamini et al., 2019](#); [Hasanah et al., 2024](#)) and covariational reasoning ([Kertil, 2020](#)) receive increasing attention, the literature does not clearly

establish whether these represent distinct constructs or components of a unified framework of mathematical reasoning.

In parallel, metaphorical thinking has been widely implemented across mathematics education, yet its role is often implicit rather than systematically theorized. Studies consistently show that metaphors function as cognitive bridges that connect abstract mathematical ideas with students' concrete experiences (Purwasi & Fitriyana, 2021). In addition, metaphors support the development of mental representations, particularly through visual and gestural forms, as demonstrated by Alcaraz-Carrión et al. (2022). The use of metaphors also appears to enhance students' engagement in higher-order thinking processes, including justification and argumentation (Hendriana et al., 2017).

Despite these promising findings, the literature tends to present metaphorical thinking in an overly instrumental manner as a teaching aid rather than as a central cognitive mechanism. Scheiner et al., (2022) emphasized that metaphors shape how students organize mathematical experiences, yet many studies do not operationalize this claim within a clear analytical framework. Similarly, Falani et al., (2023) highlighted the role of metaphors in representation and communication, but their integration into broader models of reasoning remains underexplored. This suggests that metaphorical thinking is frequently applied in practice without being fully conceptualized at the theoretical level.

Another critical limitation concerns the evaluation of impact. Although studies report positive effects of metaphor-based instruction on reasoning, most rely on performance outcomes rather than direct analysis of reasoning processes. For example, improvements in students' reasoning are often inferred from task performance (Purwasi & Fitriyana, 2021) rather than from detailed examination of how students construct arguments or generalizations. This creates a gap between observed outcomes and underlying cognitive mechanisms.

The diversity of metaphor types identified—conceptual, structural, orientational, cultural, and embodied—further illustrates both the richness and the lack of systematicity in the field. Conceptual metaphors such as “functions as machines” and embodied metaphors expressed through gestures Alcaraz-Carrión et al., (2022) are among the most frequently reported. Structural metaphors are shown to support relational understanding in advanced topics (Hicks, 2024), while cultural metaphors connect mathematical ideas with students' lived experiences (Cervantes-Barraza & Araujo, 2023). Nevertheless, existing studies rarely compare the effectiveness of these different types, nor do they examine how specific metaphors align with particular mathematical concepts or levels of reasoning.

The integration of metaphorical thinking with various instructional models such as Realistic Mathematics Education, ethnomathematics, problem-based learning, and technology-enhanced learning demonstrates its flexibility across contexts (Adelia et al., 2025; Kertil, 2020; Masfingatini et al., 2020). However, this flexibility also introduces ambiguity. Because metaphorical thinking is embedded within broader instructional approaches, its specific contribution to students' reasoning is difficult to isolate. Consequently, claims regarding its effectiveness may be overstated or conflated with the effects of the overall instructional model.

A particularly significant finding of this review is the limited integration between research on metaphorical thinking and mathematical reasoning. While both areas have developed substantially, they largely evolve in parallel rather than in an integrated manner. Studies on reasoning rarely position metaphor as a deliberate cognitive strategy (Fraihat et al., 2022; Hansen & Naalsund, 2025), whereas studies on metaphorical thinking do not consistently frame their findings within established models of mathematical reasoning. This disconnect indicates a major theoretical and empirical gap.

Furthermore, methodological limitations persist across the literature. Most studies employ short-term or cross-sectional designs, despite the fact that both metaphorical thinking and mathematical reasoning develop progressively over time. The absence of longitudinal research limits understanding of how these constructs evolve and interact across different stages of learning. In addition, disparities across educational levels, where metaphorical thinking is more commonly studied in school contexts and reasoning in higher education, further hinder a comprehensive understanding of their relationship.

Finally, although some studies incorporate digital tools, the role of technology in supporting metaphorical thinking remains underexplored. [Ghifari \(2025\)](#) shows that embedding metaphors in digital environments can enhance learning effectiveness, yet the literature has not sufficiently examined how dynamic visualizations and interactive representations can facilitate deeper reasoning processes.

Taken together, these findings suggest that future research should move beyond descriptive and fragmented approaches toward more integrated and theoretically grounded investigations. Specifically, there is a need to develop a unified framework that connects metaphorical thinking with models of mathematical reasoning, to conduct longitudinal studies that capture developmental trajectories, and to explore the potential of technology-enhanced environments in supporting metaphor-based reasoning. Such efforts are essential for advancing both the theoretical understanding and practical implementation of innovative approaches in mathematics education.

CONCLUSION

This Systematic Literature Review (SLR) provides a comprehensive synthesis of research on students' mathematical reasoning and metaphorical-thinking-based learning approaches in mathematics education from 2017 to 2025. The review demonstrates that mathematical reasoning is increasingly conceptualized as a multidimensional construct encompassing cognitive, representational, social, contextual, and collaborative dimensions. At the same time, metaphorical thinking emerges as a promising cognitive mechanism that supports reasoning by connecting abstract mathematical concepts with learners' concrete experiences through conceptual, structural, orientational, cultural, and embodied metaphors.

A key contribution of this review is the integration of two research domains that have largely developed independently. The findings indicate that metaphorical thinking has the potential to enhance mathematical reasoning by facilitating conceptual understanding, representation, argumentation, and higher-order thinking processes across various instructional models and learning environments. However, the review also reveals that empirical research explicitly examining the relationship between metaphorical thinking and mathematical reasoning remains limited. Existing studies tend to focus on learning outcomes rather than the underlying reasoning processes through which metaphors influence mathematical thinking.

The review has important implications for both research and practice in mathematics education. Theoretically, it highlights the need for a more integrated framework that positions metaphorical thinking as a central component of mathematical reasoning rather than merely an instructional aid. Practically, the findings suggest that educators may benefit from intentionally incorporating metaphor-based strategies to support students' reasoning and conceptual development. Future research should investigate the mechanisms through which different metaphor types influence specific forms of mathematical reasoning, conduct longitudinal studies across educational levels, and explore the potential of technology-enhanced environments in fostering metaphor-based mathematical reasoning.

CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this paper.

AUTHOR CONTRIBUTIONS STATEMENT

ADF: conceptualization, methodology, literature search, data curation, formal analysis, and writing—original draft preparation. SF: writing—review and editing, critical revision of the manuscript, supervision, and final approval. JAD: supervision, validation, and final approval.

AI DISCLOSURE STATEMENT

The authors used Grammarly during the preparation of this manuscript to check grammar, spelling, and language clarity. The authors reviewed and edited all suggested changes and take full responsibility for the final content of the manuscript. No AI tools were used to generate, analyze, interpret, or write the scientific content of this research.

ACKNOWLEDGMENTS

The authors would like to express their sincere gratitude to the affiliated institution for providing academic support and facilities throughout this research. The authors also appreciate the valuable feedback and constructive suggestions from colleagues and reviewers, which have contributed to the improvement of this manuscript.

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AUTHOR PROFILES



Annisa Desti Fahjri (annisadestifahjri@upi.edu), is currently a graduate student in Mathematics Education at Universitas Pendidikan Indonesia. His research interests on developing innovative instructional approaches to enhance students’ higher-order thinking skills. Google Scholar: C30ef2EAAAAJ, Orcid: 0009-0002-7971-7604



Siti Fatimah (sitifatimah@upi.edu) is a Professor in Mathematics Education at Universitas Pendidikan Indonesia. Her academic expertise lies in **dynamical systems and mathematical analysis**, with a strong research focus on mathematics education, particularly in developing students’ higher-order thinking skills and mathematical reasoning. Google Scholar: PvsWmS4AAAAJ, Sinta ID: 5978161, Orcid: 0000-0002-4379-9540, Scopus links: <https://www.scopus.com/authid/detail.uri?authorId=58765794800>



Jarnawi Afgani Dahlan (jarnawi@upi.edu) is a Professor in Mathematics Education at Universitas Pendidikan Indonesia. His research interests include **models, strategies, methods, and approaches in mathematics learning**, with a particular emphasis on enhancing students’ mathematical reasoning and problem-solving abilities. Google Scholar: TBbpIxAAAAAJ, Sinta ID: 5974544, Orcid: 0000-0002-9290-7755, Scopus links: <https://www.scopus.com/authid/detail.uri?authorId=57193867557>